Draft version December 15, 2018
Typeset using LaTeX **modern** style in AASTeX62

The Sirens of August: Detection of Five Gravitational Wave Events in August 2017 Is Consistent With A Constant Rate

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August 2017 was a happy time for gravitational wave astronomy. Of the 11 gravitational waves—10 binary black hole (BBH) mergers and one binary neutron star (BNS) merger—detected to date by the Advanced LIGO (Aasi et al. 2015) and Advanced Virgo (Acernese et al. 2015) instruments, five occurred in August 2017 (The LIGO Scientific Collaboration & the Virgo Collaboration 2018): GW170809, GW170814 (Abbott et al. 2017a), GW170817 (Abbott et al. 2017b), GW170818, and GW170823. Throughout LIGO's second observing run (O2) there were $(1, 0, 0, 0, 1, 1, 5)$ gravitational waves from merging compact binary systems detected in January, February, March, April, June, July, and August 2017 (we omit May because it comprises negligible observing time; O2 finished in August 2017). In LIGO's first observing run (O1) there were $(1, 1, 1)$ compact binary detections in September, October, and December 2015 (we omit November because it also comprises negligible observing time). Here we show that the outlier of five detections in August is nonetheless consistent with a constant monthly detection rate. A constant spacetime merger rate density is the starting point of merger rate analyses in The LIGO Scientific Collaboration & the Virgo Collaboration (2018) and The LIGO Scientific Collaboration & The Virgo Collaboration (2018); given the $\sim 5\,\mathrm{Gyr}$ of cosmic time probed by the O2 observations and the $\sim 10\,\mathrm{Myr}$ lower limit on the stellar evolution timescale of even the most massive BBH progenitor systems, fluctuations in the detection rate would be overwhelmingly likely to be of terrestrial origin and could indicate un-modeled systematic effects in the instruments.

We ignore effects at the $\sim 10\%$ level that could affect the detection rate, such as: the varying length of calendar months, variations in detector sensitivity over O1 and

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O2, that the sensitivity in O1 is about 3/4 that of O2, variations in duty cycle over O2, the addition of Advanced Virgo to the detector network in August 2017, etc. There were eleven detections in ten months. The probability that a Poisson process with a mean rate of 11/10 per month will have five or more events in one month is $\simeq 0.54\%$. Over ten months, the probability that at least one month will have five or more detections is $\simeq 5.3\%$. Thus it is not particularly surprising that we see one month with five detections.

The above simplistic analysis is but one choice from an entire menu of possible calculations. For example, we could ask whether the detections in August are consistent with the observed rate from the previous six months' of data taking (answer: much less so); or we could ask whether the detections in the second half of the combined O1 and O2 runs are consistent with the rate from the first half (answer: much more so). Or.... Most calculations from this menu will arrive at probabilities in the 1–10% range. The simplistic analysis above also ignores that we have statistical uncertainty in the mean rate; 11/10 detections per month is the most likely value, but the statistical uncertainty is $\sim 1/\sqrt{11} \simeq 30\%$. This uncertainty will also vary as the data are grouped differently in each possible calculation discussed above.

An elegant way to simultaneously address the multiple comparisons problem and rate measurement uncertainty is through multilevel modeling[1] (Gelman & Hill 2006). The starting point for a multilevel analysis is an independent fit of a Poisson rate to each months' data:

$$n_i \sim \text{Poisson}\left(\Lambda_i\right), \tag{1}$$

where n_i is the number of detected events in month i and Λ_i is the Poisson mean rate of detections in that month. At this point each months' rate is treated independently of the other months; this is certainly not an optimal model, since there is certainly *some* month-to-month correlation. If we impose that the Λ_i are drawn from a common distribution, say a normal:

$$\Lambda_i \sim N(\mu, \sigma) \tag{2}$$

then the parameter μ measures the mean of the monthly detection rate and σ measures the amount of month-to-month scatter. The posterior distribution for the Λ_i, μ, and σ given the monthly event counts n_i is therefore

$$\pi\left(\Lambda_i, \mu, \sigma \mid n_i\right) \propto \left[\prod_{i=1}^{N_\text{month}} \frac{\Lambda_i^{n_i}}{n_i!} \exp\left[-\Lambda_i\right] \phi\left(\Lambda_i \mid \mu, \sigma\right)\right] p\left(\mu, \sigma\right), \tag{3}$$

where $p\left(\mu, \sigma\right)$ is the prior we impose on μ and σ and $\phi\left(x \mid \mu, \sigma\right)$ is the probability density for a Gaussian with mean μ and standard deviation σ evaluated at x.

In this model, setting $\sigma \equiv 0$ forces a constant monthly rate; setting $\sigma \to \infty$, we recover independent fits to each months' data. Reality is somewhere in between; the

[1] Multilevel modeling is sometimes also called hierarchical modeling or population analysis. In an astrophysical context, see Hogg et al. (2010), Mandel (2010), or, applied to gravitational waves, The LIGO Scientific Collaboration & The Virgo Collaboration (2018).

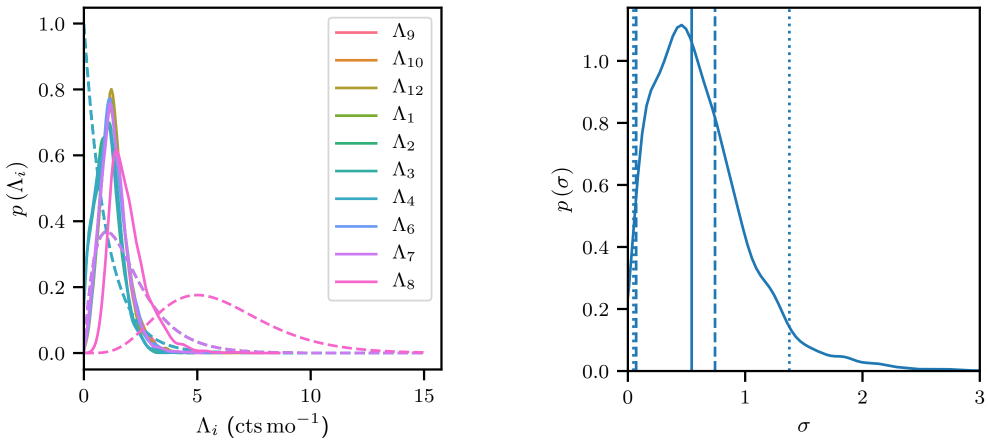

Figure 1. Inference on the per-month detection rate and the scatter parameter describing the population of per-month rates. Left: the posterior on the mean per-month detection rate, Λ, for each calendar month of O1 and O2 with significant observing time. The curves are labeled by the month index (1 = January, 2 = February, etc). The August outlier is apparent in the posterior for Λ_8. Dashed curves give the posterior for a flat prior imposed on each of the Λ_i; solid curves give the posterior under the multilevel log-normal prior. It is apparent that this latter "shrinks" the monthly estimates toward a common rate. For both the independent and pooled analyses there is some overlap in the posteriors, suggesting that a constant monthly detection rate is consistent with our inferences. Right: the posterior on the parameter σ that controls the scatter in the population of (log) detection rates. We obtain $\sigma = 0.54^{+0.20}_{-0.47}$ (median and 68% CI); the solid line indicates the median value, dashed lines the 68% CI, and dotted lines the 95% CI (which includes $\sigma = 0$).

key insight of multilevel modeling is that we can allow the *data* to inform the value of σ and the degree of "shrinkage" (e.g. Lieu et al. 2017) or "pooling" of the data in the fit[2]. The multilevel model effectively explores all possible pooled combinations of the data as σ varies, and chooses those that are consistent with the observed month-to-month scatter, while accounting for the multiple comparisons and the varying accuracy of the overall rate estimation with various poolings.

The result of a fit for the eleven Λ_i and μ and σ to the number of detections by month is shown in Figure 1. We used broad $N(0, 10)$ priors for both μ and σ. Such broad priors have negligible effect on the posterior given the precision of our measurements of μ and σ. The partially-pooled posteriors for the monthly rates Λ_i overlap significantly, suggesting that a constant monthly detection rate is consistent with our data set. This is confirmed by the posterior for the month-to-month scatter in the rate, which includes $\sigma = 0$ in its 95% ("2σ") credible interval. We therefore conclude that five detections in August are consistent with a constant monthly detection rate throughout O2.

[2] Eq. (2) corresponds to imposing a white Gaussian process prior on the log detection rate. A natural extension would be to introduce a temporal correlation length (Foreman-Mackey et al. 2014; Kelly et al. 2014; Foreman-Mackey et al. 2017; Farr et al. 2018) in the prior so that the rate variations have a degree of smoothness from month-to-month. But at this point we are probably getting too far out ahead of our small data set....

This document is LIGO Technical Report Number T1800529. A `git` repository containing analysis code and this LaTeX document can be found at https://git.ligo.org/will-farr/TheSirensOfAugust.

Software: matplotlib (Hunter 2007), seaborn https://doi.org/10.5281/zenodo.1313201, pymc3 (Salvatier et al. 2016)

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