Numerical Relativity for Vacuum Binary Black Hole

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Outline

- What is numerical relativity?
- What is its role in the detection and interpretation of gravitational waves?

What is numerical relativity for binary black holes?

- 3 stages for binary black hole coalescence:
    - inspiral
    - merger
    - ringdown
- Inspiral waveform: Post-Newtonian approximations
- Ringdown waveform: perturbation theory

- **Merger of 2 black holes:**
  - extremely energetic, nonlinear, dynamical and violent
  - the strongest GW signal
  - warpage of spacetime
  - strong gravity regime
- **The Einstein equations**: $$G_{\mu\nu} = 8\pi T_{\mu\nu}$$
  - encodes all of gravity
  - 10 equations which would take up ~100 pages if no abbreviations or simplifications used (and if written in terms of the spacetime metric alone)
  - pencil and paper solutions possible only with spherical (Schwarzschild) or axisymmetric (Kerr) symmetries.
- **Vacuum:** $$G_{\mu\nu} = 0$$

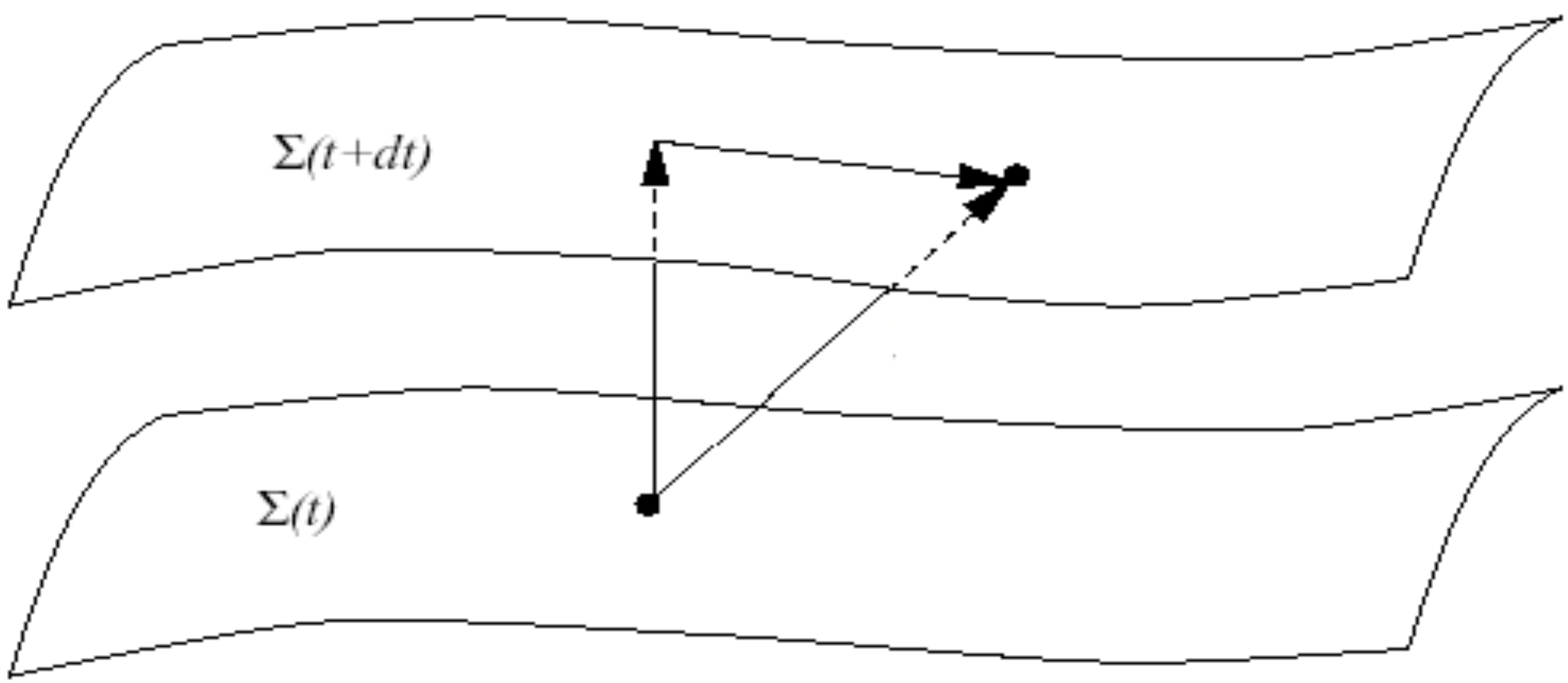
  (empty space with no matter)

Numerical Relativity is:

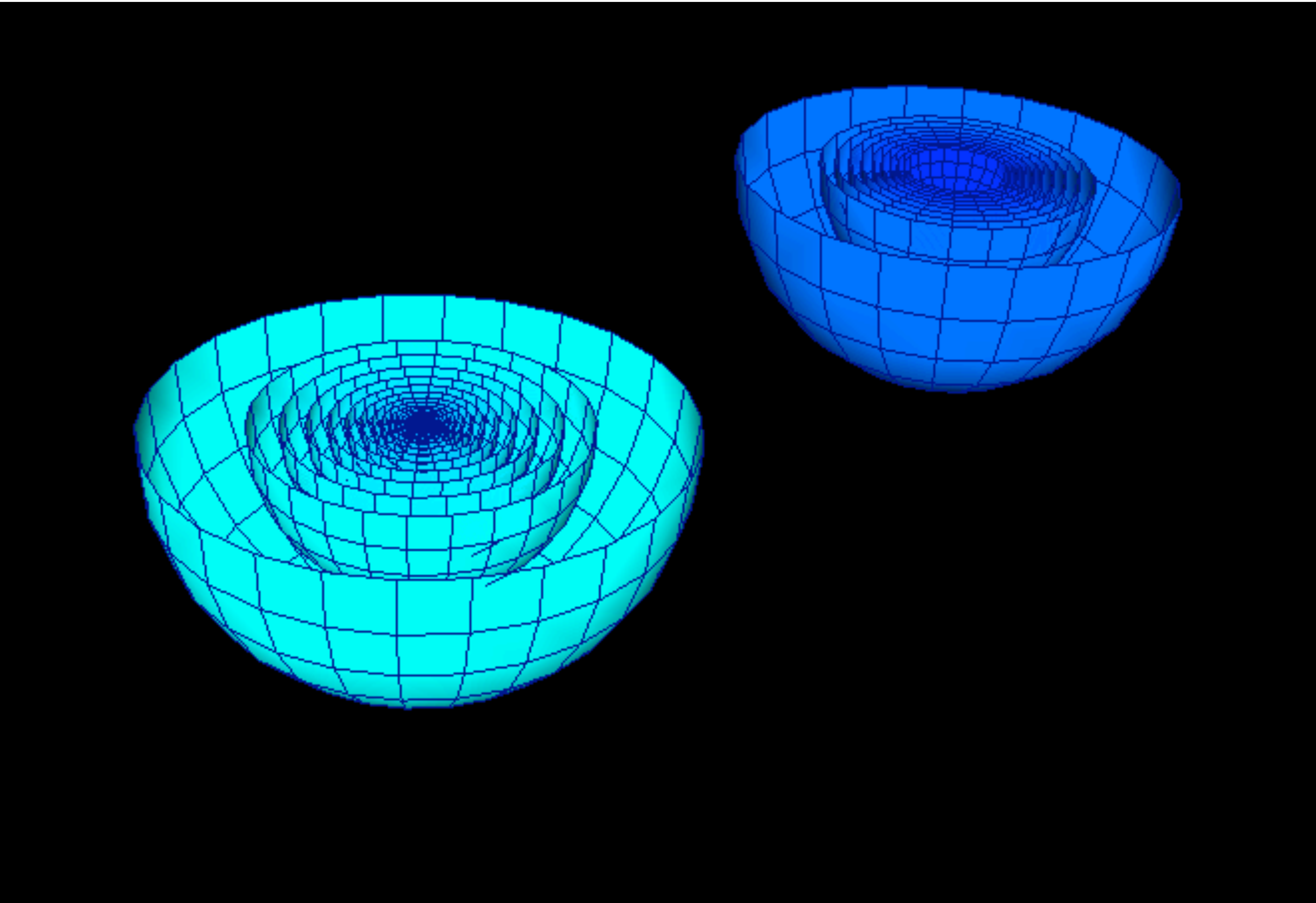
- Directly solving the full dynamical Einstein field equations using high-performance computing.
- A **very** difficult problem:
    - the first attempts at numerical simulations of binary black holes was in the 1960s (Hahn and Lindquist) and mid-1970s (Smarr and others).
    - the full 3D problem remained unsolved until **2005** (Frans Pretorius, followed quickly by 2 other groups- Campanelli et al., Baker et al.).

Solving Einstein's Equations on a Computer

- 3+1 approach (Arnowitt-Deser-Misner):
  - Slice 4D spacetime into 3D-spacelike hypersurfaces
  - Solve on one slice of space, then step forward in time



Evolution: domain decomposition



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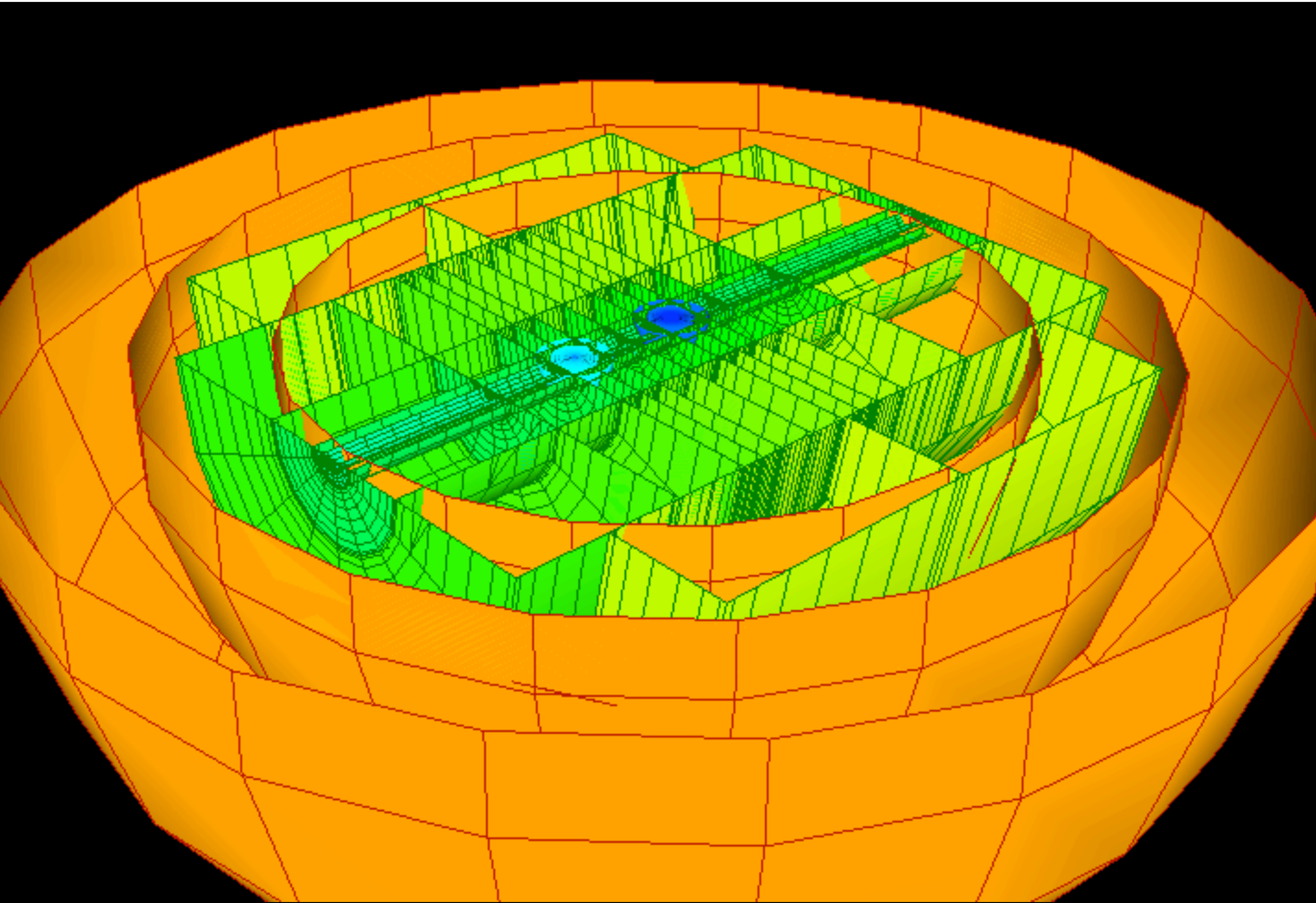
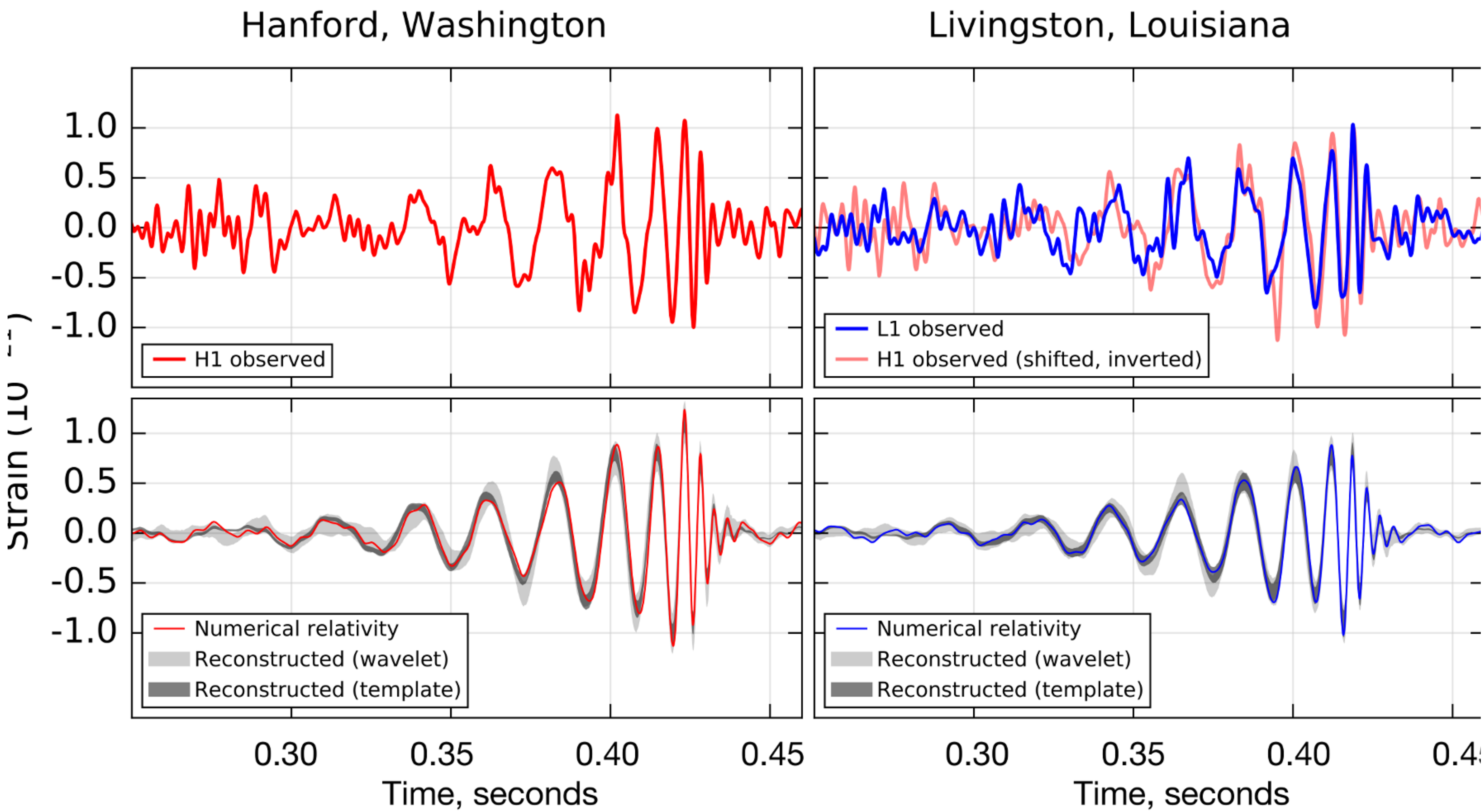


Role of Numerical Relativity

- **detection**: search for binary black hole coalescence in noisy detector data requires accurate waveforms (although GW150914 could be seen by eye)
- **interpretation**:
  - inferring the properties of the binary black hole system which created the signal
  - and of the remnant black hole left after the merger: its mass, spin and recoil kick velocity (which can cause it to be ejected from the host galaxy)
- All BBH observations to date by LIGO and VIRGO have been analyzed using waveform families constructed from numerical relativity



GW150914 LIGO Hanford and Livingston Sept 14, 2015

B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. **116**, 061102 – Published 11 February 2016

The numerical relativity waveforms were computed by the SXS collaboration.

From the SXS website: black-holes.org:

"The fit is beautiful. It tells us that the source of the gravitational waveform was a pair of black holes, spiraling together, colliding, and merging into a single black hole, with one hole 36 times heavier than the sun, the other 29 times heavier than the sun, and the final hole 62 times heavier than the sun. The change from 29+36=65 solar masses to 62 solar masses tells us that the gravitational waves carried away three solar masses of energy. […] And from the SXS simulation, we learn that during the very brief collision itself, the power (the energy per unit time) being injected into the gravitational waves was greater than the total power emitted by all the stars in the universe put together."

https://youtu.be/c-2XIuNFgD0

Some Closing Words

- It is too expensive (computationally and humanly) to perform the millions of accurate binary black hole numerical relativity simulations necessary to cover the vast 7-dimensional parameter space (the mass ratio of the two black holes, and the two spin vectors).
- So, numerical relativity simulations are used to calibrate analytic effective one-body or phenomenological waveform models.
- They are also used to construct surrogate waveforms using reduced order modeling.