Discovering the Underlying Distributions of Black Hole Populations

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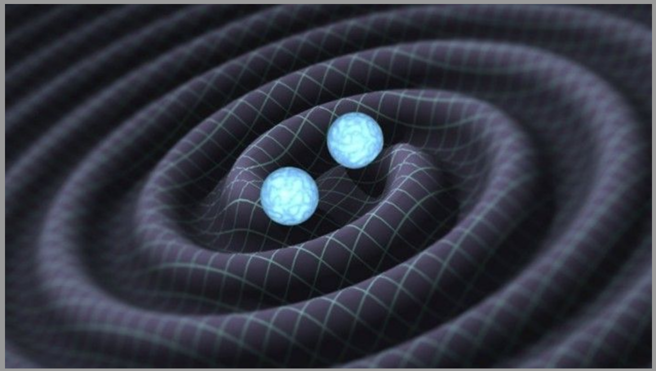


Caltech / LIGO

Overview

- Introduction to rates and populations
- Motivations
- Proposed mass distributions
- Recovering the mass distributions
- Conclusions / future work

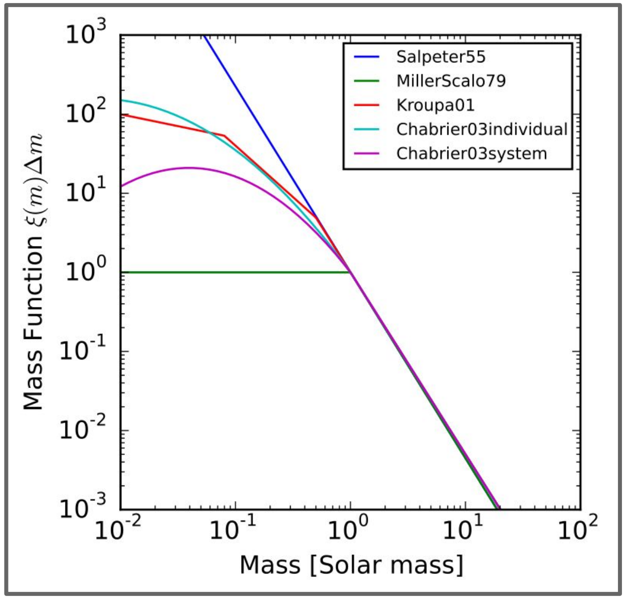
Introduction to rates and populations

- Detector sensitivity ↑, distance we can hear GWs from compact binary mergers ↑
- # of events detected will dramatically increase in the near future
- We expect tens, hundreds, or thousands of events
- Measure event rate density (in units of mergers/time/volume) as a function of mass, spin, and redshift (ignore spin for now)

$$\mathcal{R}(m_1, m_2, z) = \frac{dN}{dV_c dt_s}$$

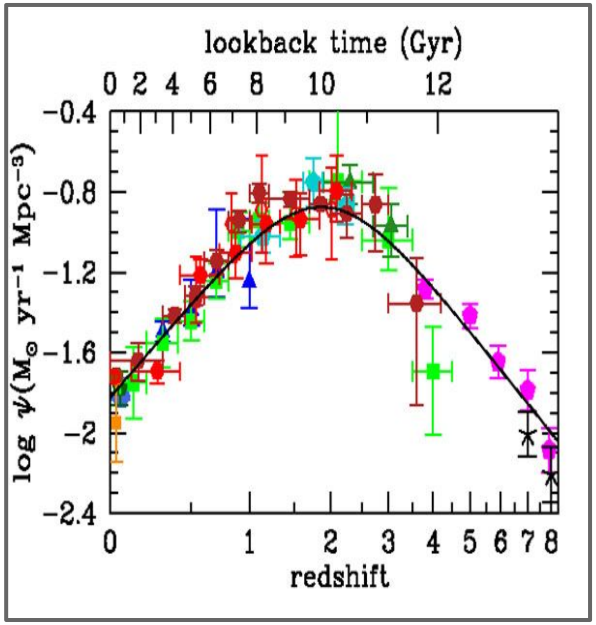
- Now is the optimal time to develop tools to use for this

Models of $R(m_1, m_2, z)$ for Single Stars



Salpeter IMF, shown in **dark blue**, on a log scale. Describes the initial mass distribution for a stellar population. It appears linearly as its true nature is a power law.

(Johannes Buchner. "Initial Mass Function," Wikipedia.)

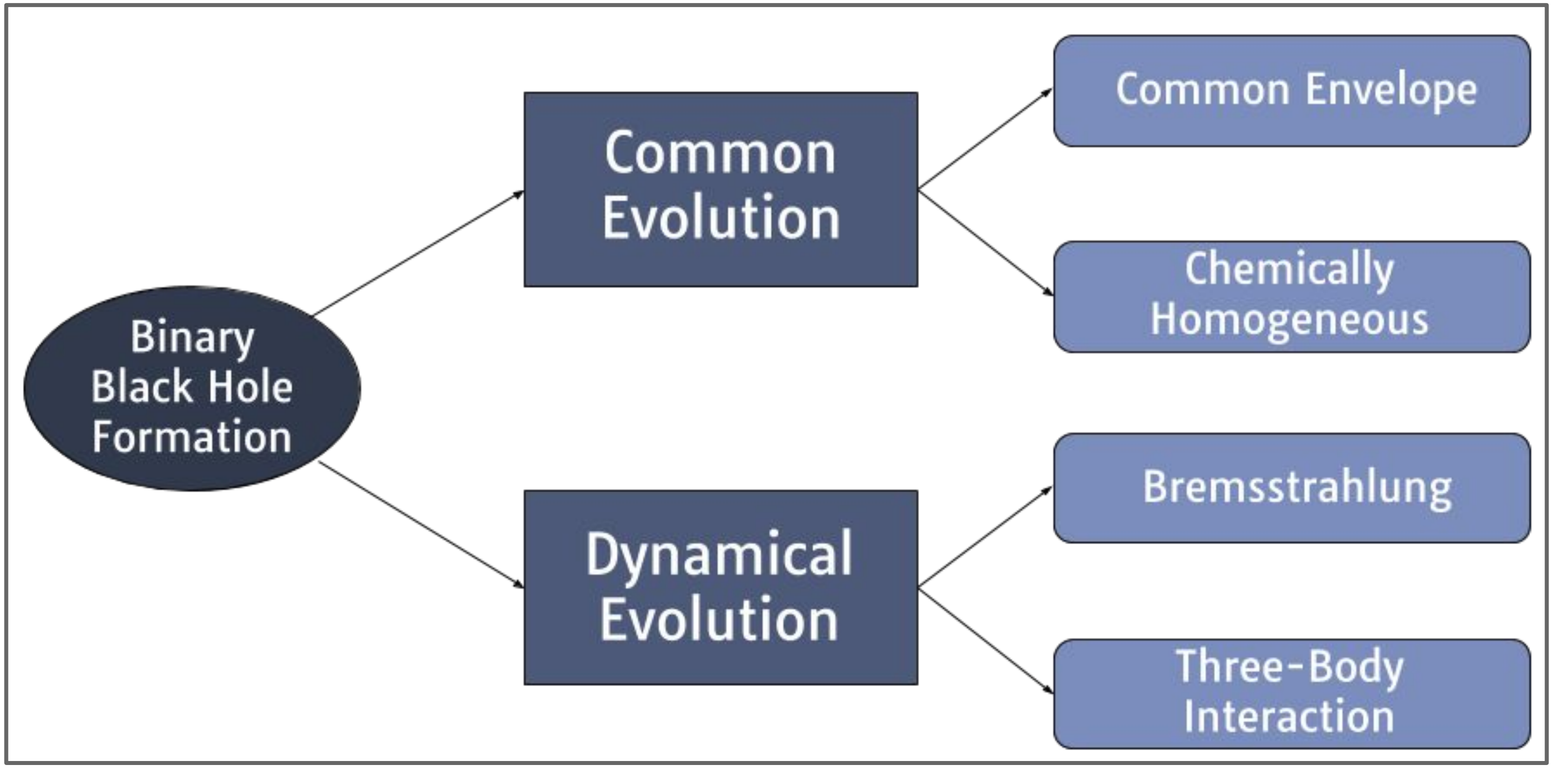


Madau-Dickinson star formation rate density (SFR/unit volume) as a function of redshift. The data points come from many other bodies of work. This distribution will be shifted left for BHs.

(Piero Madau, Mark Dickinson. *Cosmic Star Formation History*. 2014)

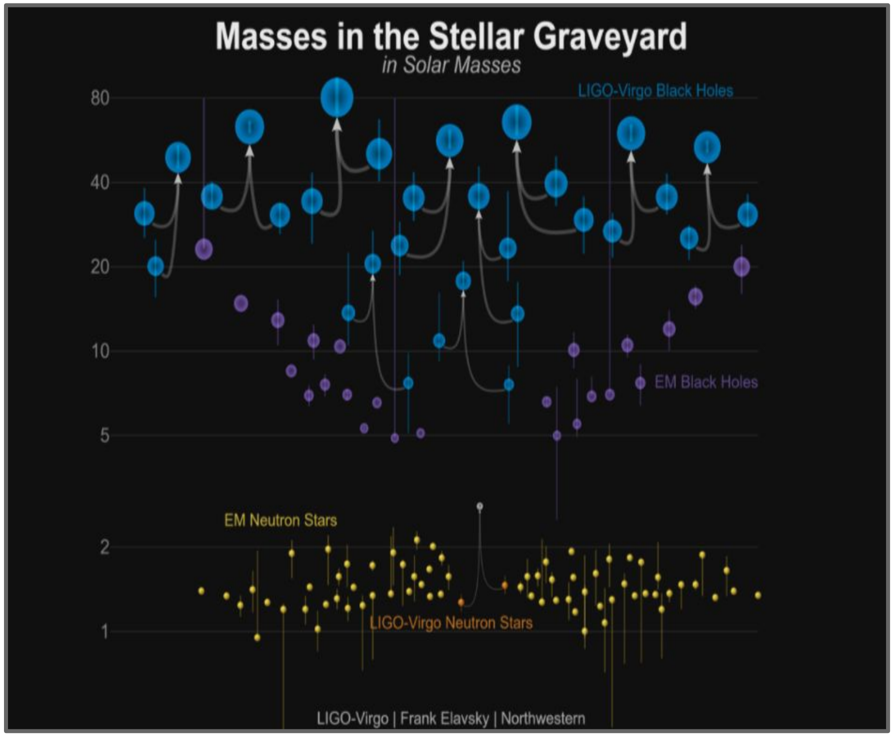
Connecting Single Stars to BBHs

Channels of BBH Formation



Populations, Mass Distributions, and Mass Gaps

- ~50-150$M_{\odot}$: Pulsational pair-instability supernovae (blows away significant portion of mass)
- ~2-5$M_{\odot}$: Possible disparity between NS and BH masses
- < 1$M_{\odot}$: Small likelihood that traditional stellar collapse would form BHs



Mathematical Process

$$\frac{d\dot{N}(\lambda)}{d\vec{\theta}} = \mathcal{R}(1+z)^{\gamma} f(m_1|\alpha) f(m_2|\beta) \frac{dV_c}{dz} \frac{dt_d}{dt_s} \frac{1}{T_d} \quad (1)$$

Naturally-occurring event rate as a function of θ = m_1, m_2, z, λ = α, β, γ

$$\hat{N}_{true} = \int \frac{dN(\lambda)}{d\vec{\theta}} d\vec{\theta}, \quad (2)$$

Naturally-occurring, true number of events

$$\hat{N}_{det} = \int \frac{dN(\vec{\lambda})}{d\vec{\theta}} \mathcal{E}(\vec{\theta}) d\vec{\theta}, \quad (3)$$

Observed number of events

$$P(N|\hat{N}_{det}, \vec{\lambda}) = \frac{\hat{N}_{det}^{N} e^{-\hat{N}_{det}}}{N!} \quad (4)$$

Probability of true # of events given detected # of events

Mathematical Process

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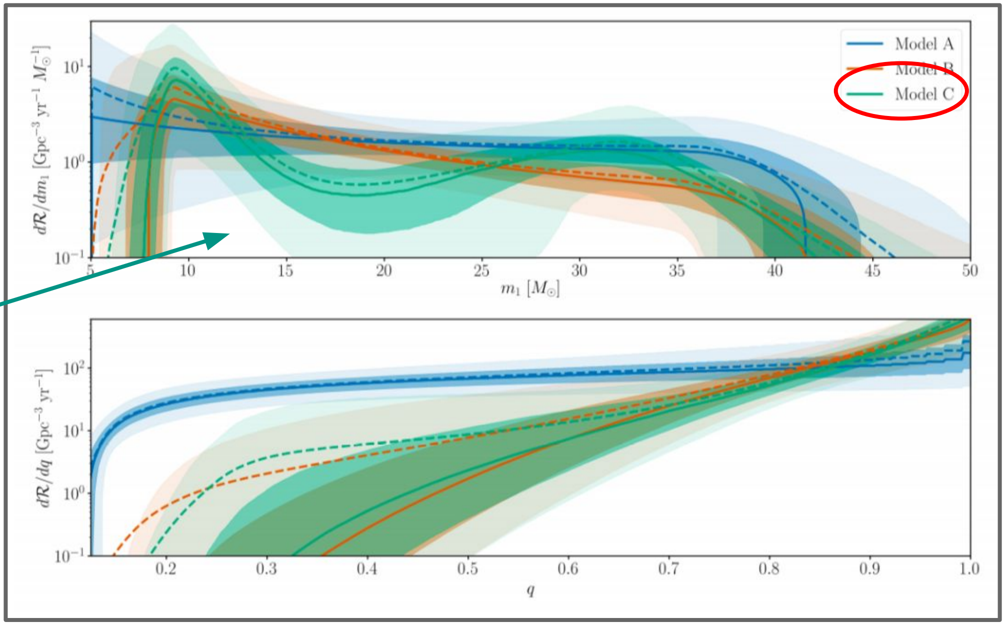
Primary Mass Distribution

Power law component



Abbott, B. P., Abbott, R., Abbott, T. D., Abraham, S., Acernese, F., Ackley, K., ... & Agathos, M. (2019). Binary Black Hole Population Properties Inferred from the First and Second Observing Runs of Advanced LIGO and Advanced Virgo. *arXiv preprint arXiv:1811.12940.*

Analysis Process

Generate optimal waveforms based on mass distributions in my model

→

Retrieve the optimal SNR from each waveform, evaluate SNR threshold

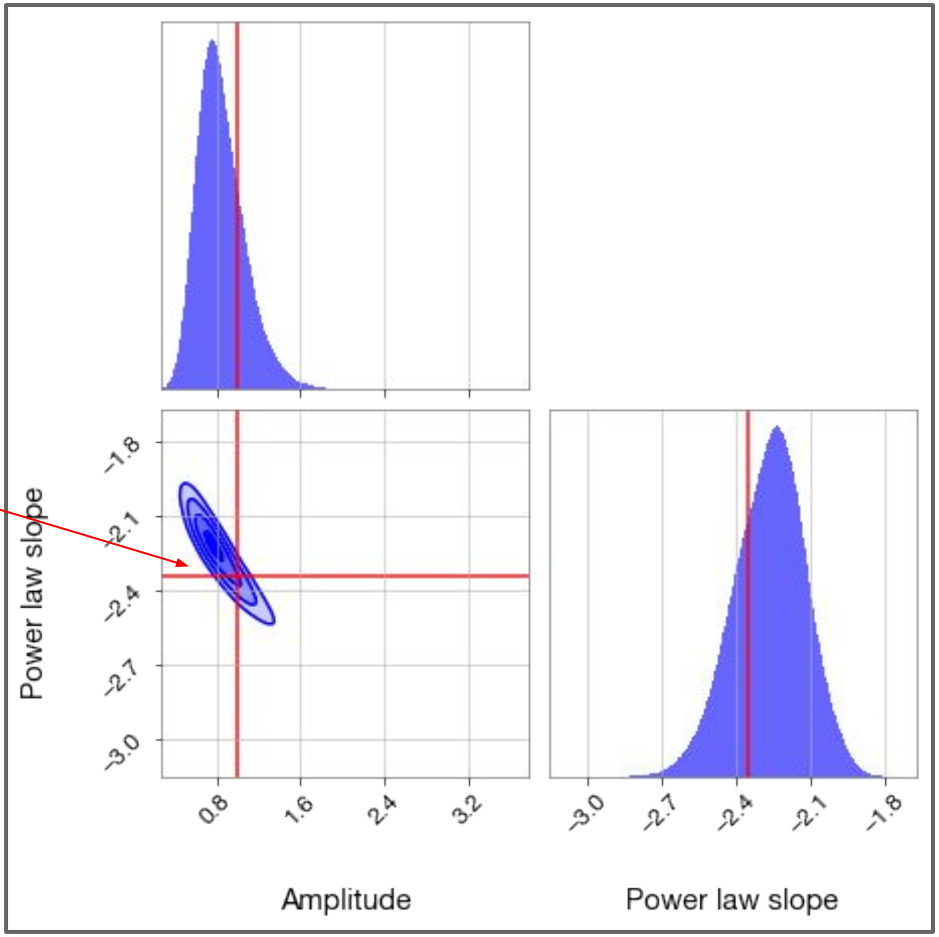
→

Parameter estimation on the retrieved mass distributions to recover the hyperparameters α, β, γ

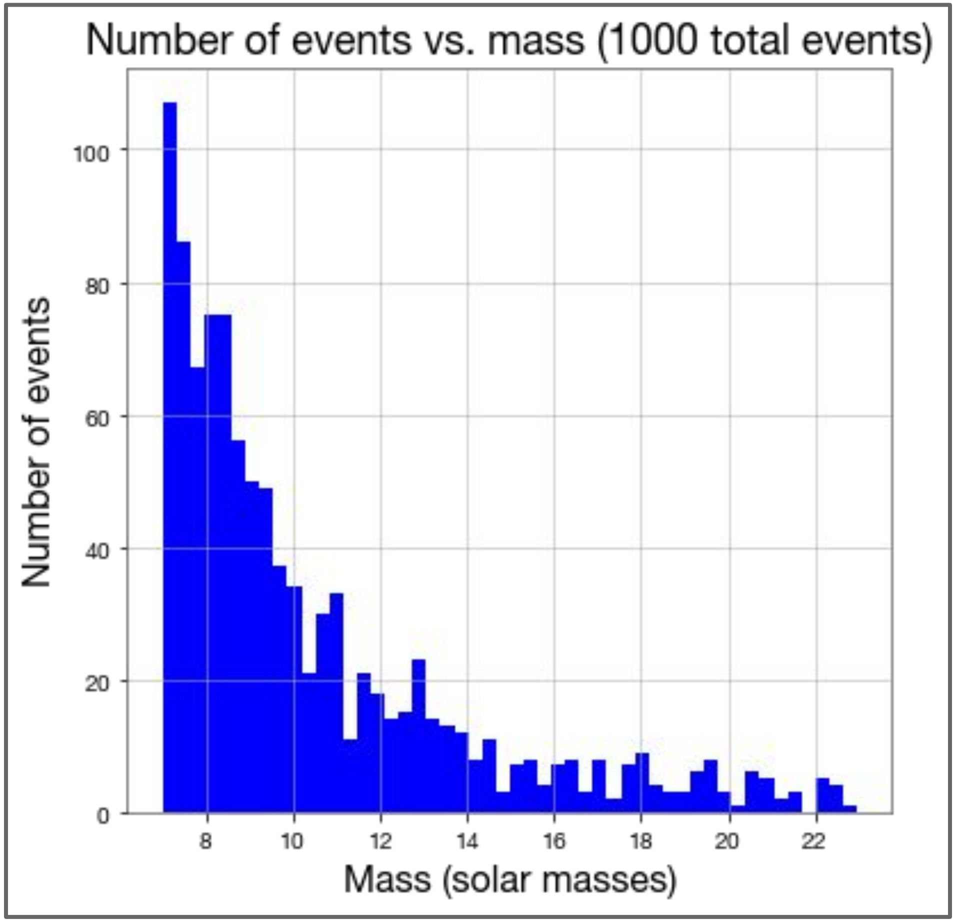
$$\rho^2{}_{opt} = \int \frac{\hat{h}^*(f)h(f)}{S(f)}df$$

Initial Parameter Estimation: Salpeter IMF

Well-recovered!



Model C Power Law Component



Parameter estimation to follow soon…

Conclusions / Future Work

- The known hyperparameters for the initial power law testing were well-recovered
- In the future, we will likely be able to conduct a similar parameter estimation on real populations of BBHs, and will be able to recover the underlying distribution

- Finishing Model C analysis / other more complex models
- Incorporating efficiency of detection
- Errors in the values of mass

Acknowledgements

- Alan Weinstein
- Jonah Kanner
- Liting Xiao
- Tom Callister
- Shreya Anand
- Ryan Magee

Other Slides

Bayesian Inference

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{\mathcal{Z}}$$

→ Bayes' Theorem

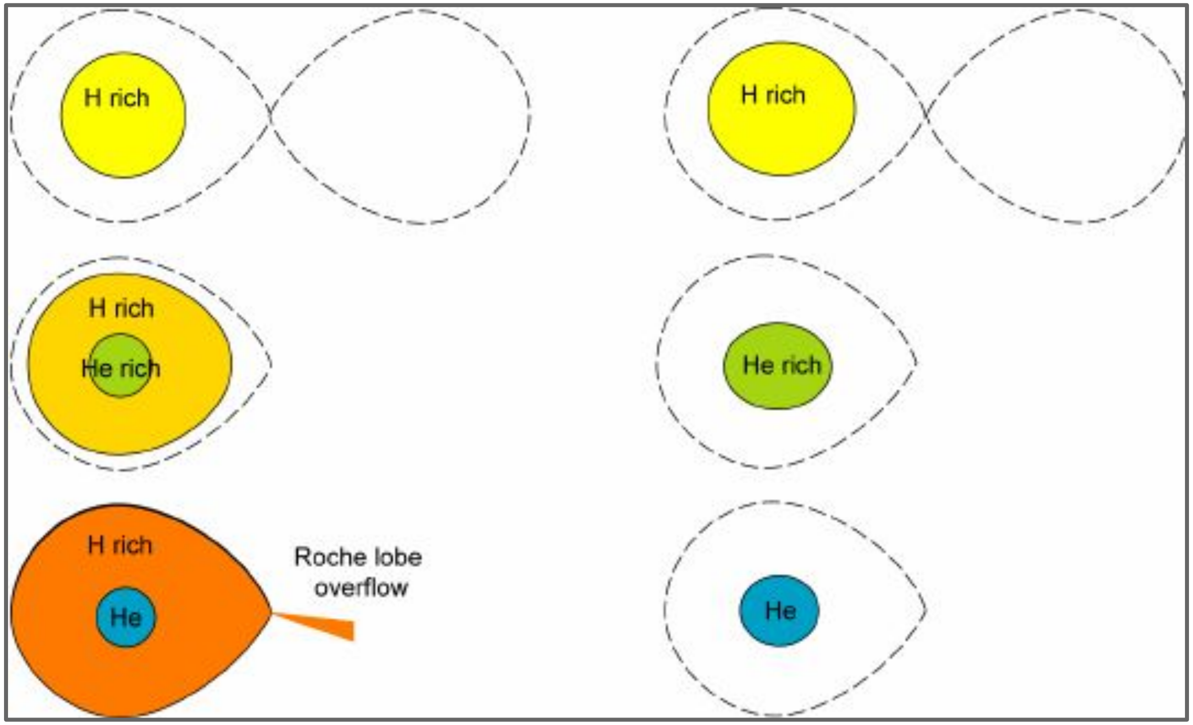
$$\mathcal{Z} \equiv \int d\theta \mathcal{L}(d|\theta)\pi(\theta)$$

→ Evidence

$$BF_2^1 = \frac{\mathcal{Z}_1}{\mathcal{Z}_2}$$

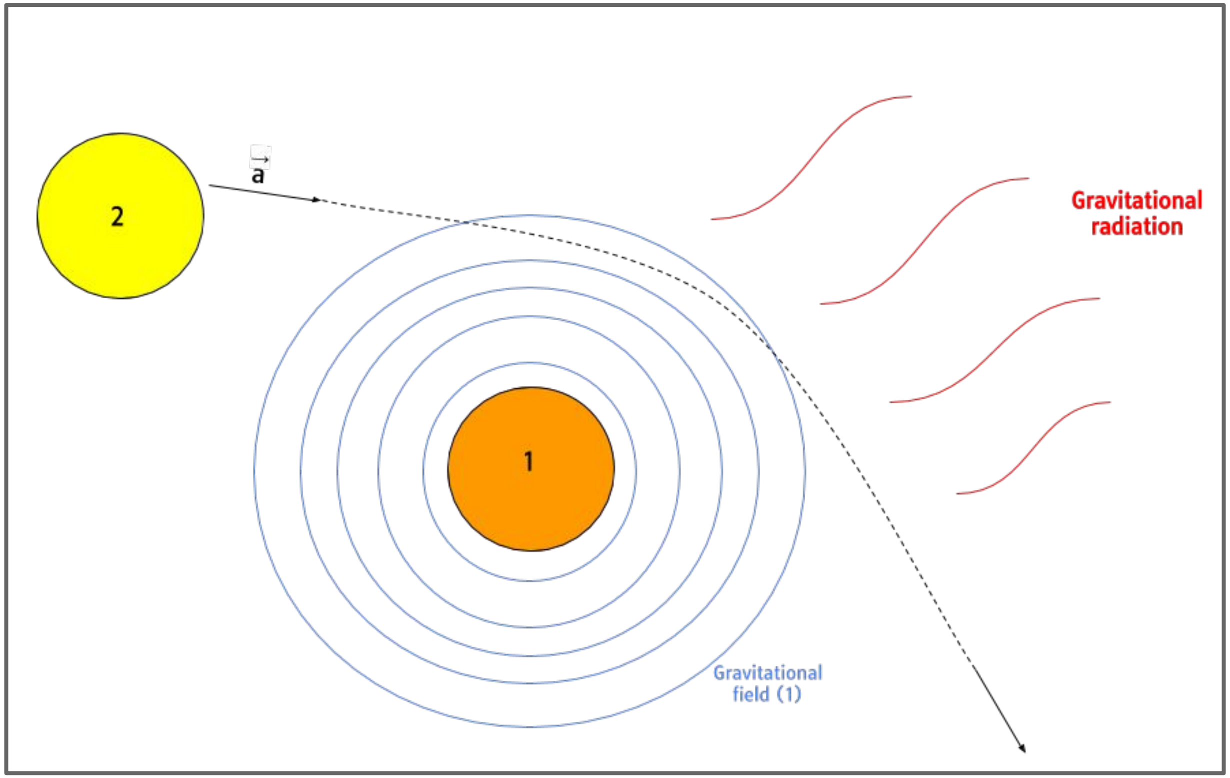
→ Bayes Factor

Common Envelope vs. Chemically Homogeneous

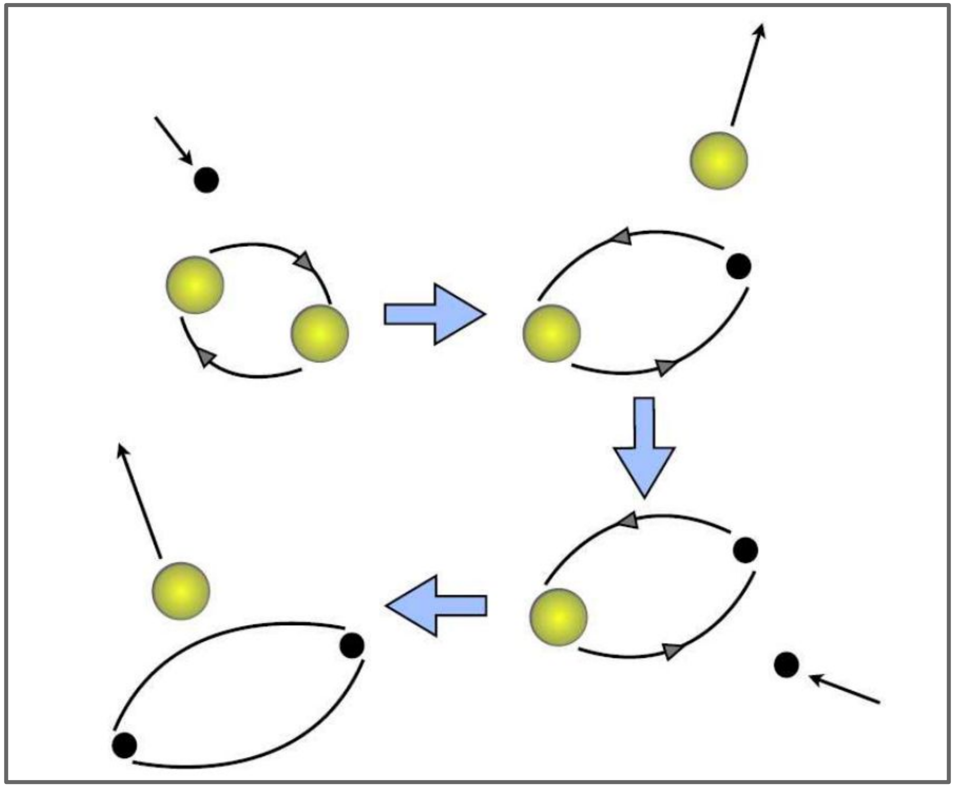


(de Mink, 2008)

Bremsstrahlung



Three-Body Interaction



(Banerjee, 2016)