Acoustic Loss Tomography in High-Q Mechanical Resonators

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LIGO-T1900391-v1

Overview

- Q factor and loss angle
- Silicon in LIGO
- GeNS experimental setup
- Measurements
  - Ringdown and amplitude locked loop
  - Signal processing and optimization
- Loss tomography
  - COMSOL: Strain energy ratio
  - Matrix inversion
  - Final results

Q factor and loss angle

- Q factor describes the resonance of an underdamped harmonic oscillator
- A higher *Q* indicates a lower rate of energy loss relative to the stored energy of the resonator
  - Oscillations decay more slowly

Internal friction ↓ Quality factor ↑

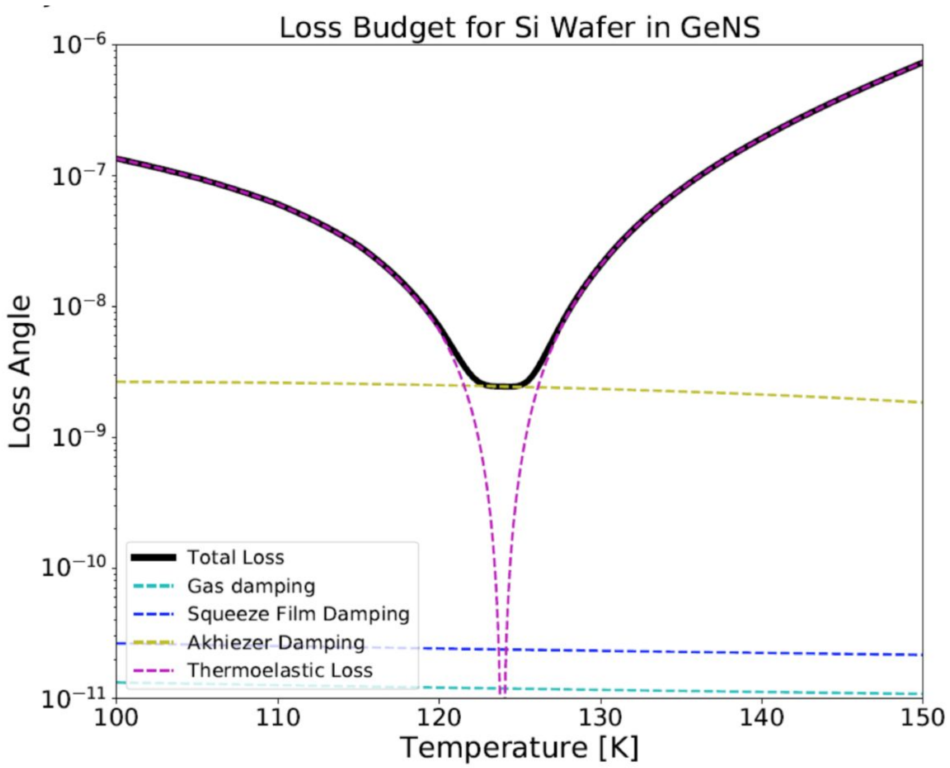
$$Q = 2\pi f \frac{\text{energy stored}}{\text{power dissipated}} = \pi f \tau$$

- Loss angle (ϕ, dissipation factor) is the loss-rate of energy of a mode of oscillation

$$\phi = \frac{1}{Q}$$

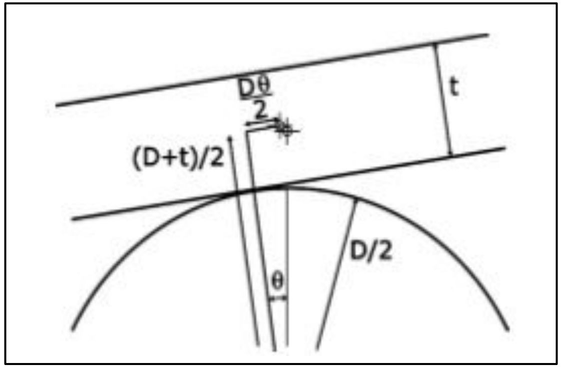
Silicon in LIGO

- LIGO Voyager - 200 kg high purity single crystalline silicon mirrors
- Thermal crossing 123K - intrinsic loss goes to 0
- Decreasing loss in silicon allows for better measurements of thin film mirror coating losses - can determine best coating options for Voyager update

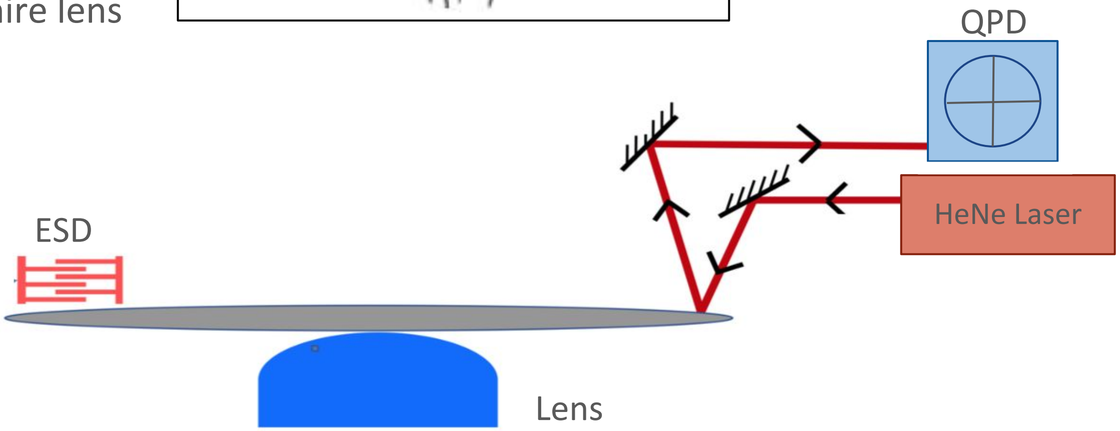


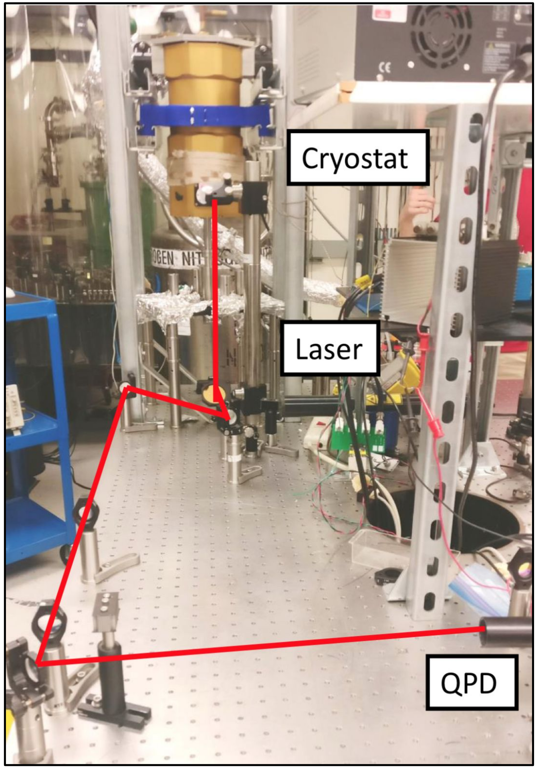
Gentle Nodal Suspension System - Cryogenic

GeNS system - single point contact between the center of the disk and sapphire lens



An actuator moves the lens to excite modes. A beam hits the edge of the disk and the motion is tracked on the QPD.

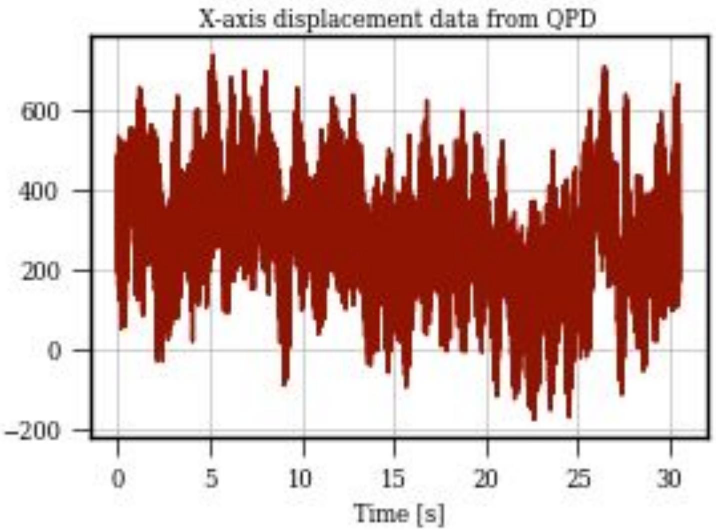


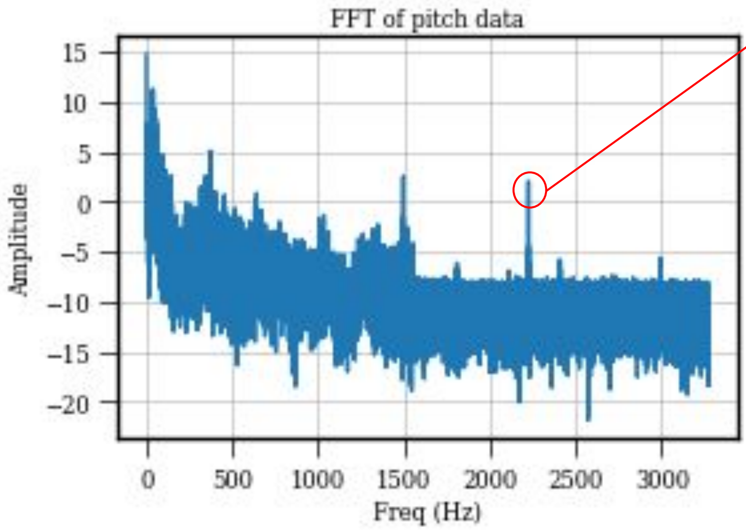


LIGO

Q measurements

- Use butterworth band-pass filter to excite different modes.
- Once a mode is excited the excitation can be turned off, and the data from a quadrant photodiode (QPD) can be analyzed to find the power spectrum density (PSD) and decay of the mode.

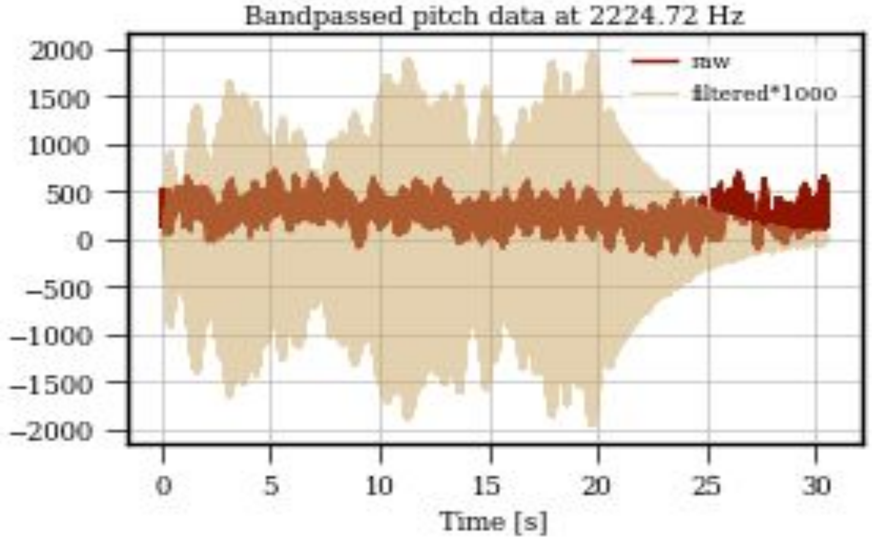


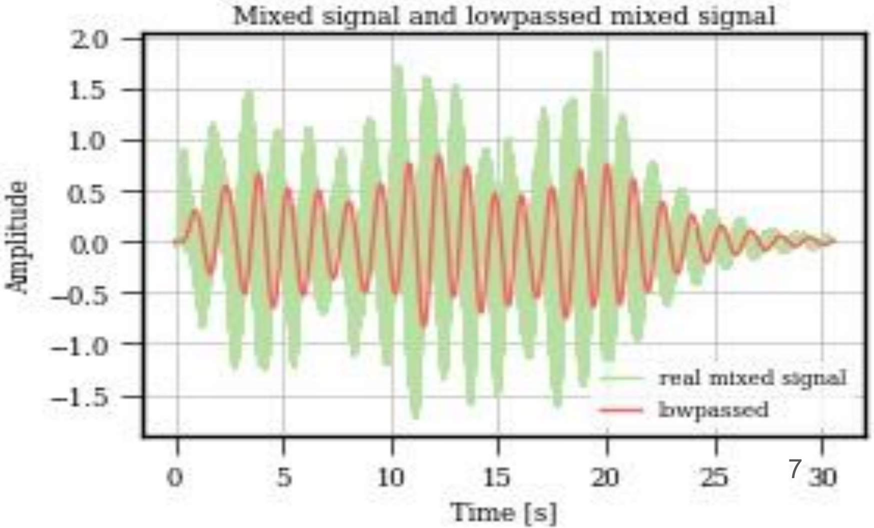


Eigenfrequency of excited mode: 2224.72 Hz

Heterodyning

- Bandpass the pitch QPD data at the known eigenfrequency
- Create new frequencies in the signal by mixing in a signal at $f_{mode} + C$, beat frequencies at $f_{mode} \pm f_{mixing}$
- Low pass the mixed signal to only keep the lower beat frequency

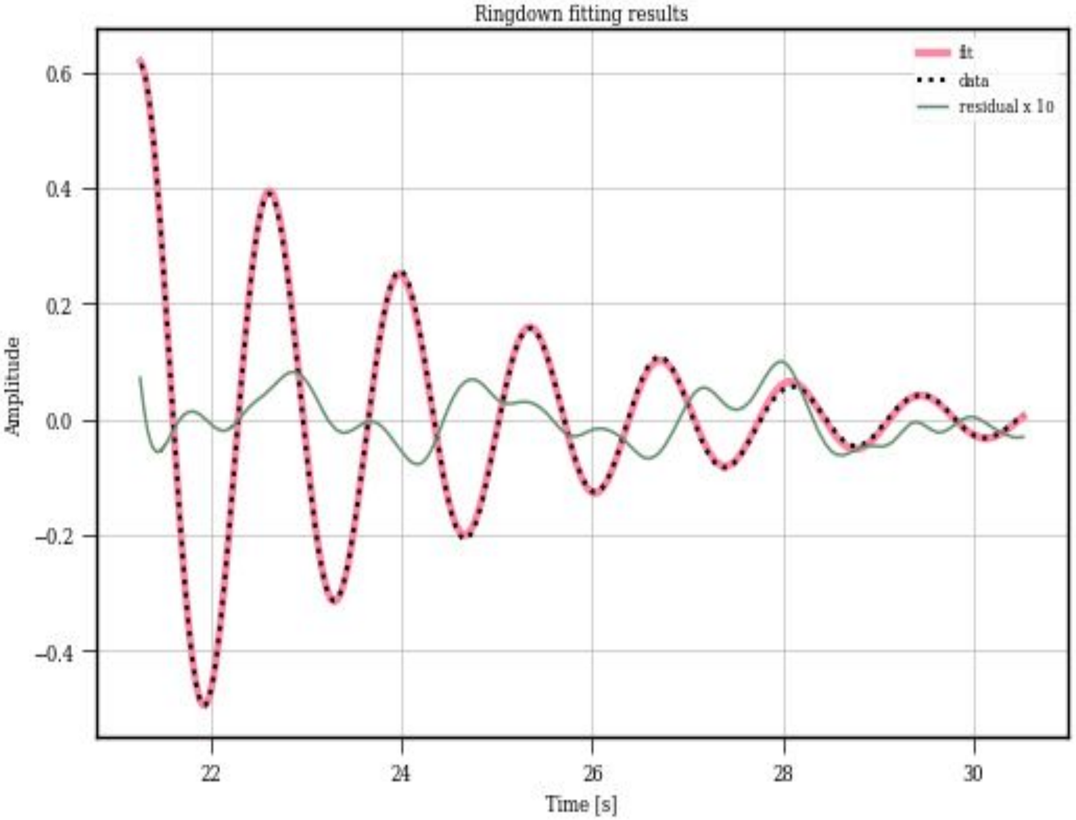




Fitting and optimization

$$f(t) = A\sin(\omega t + \phi)e^{t/\tau} + C$$

- Fit the exponentially decaying sine wave
- C, the constant offset of the mixed signal and the time interval used for the fit were varied to minimize variance.

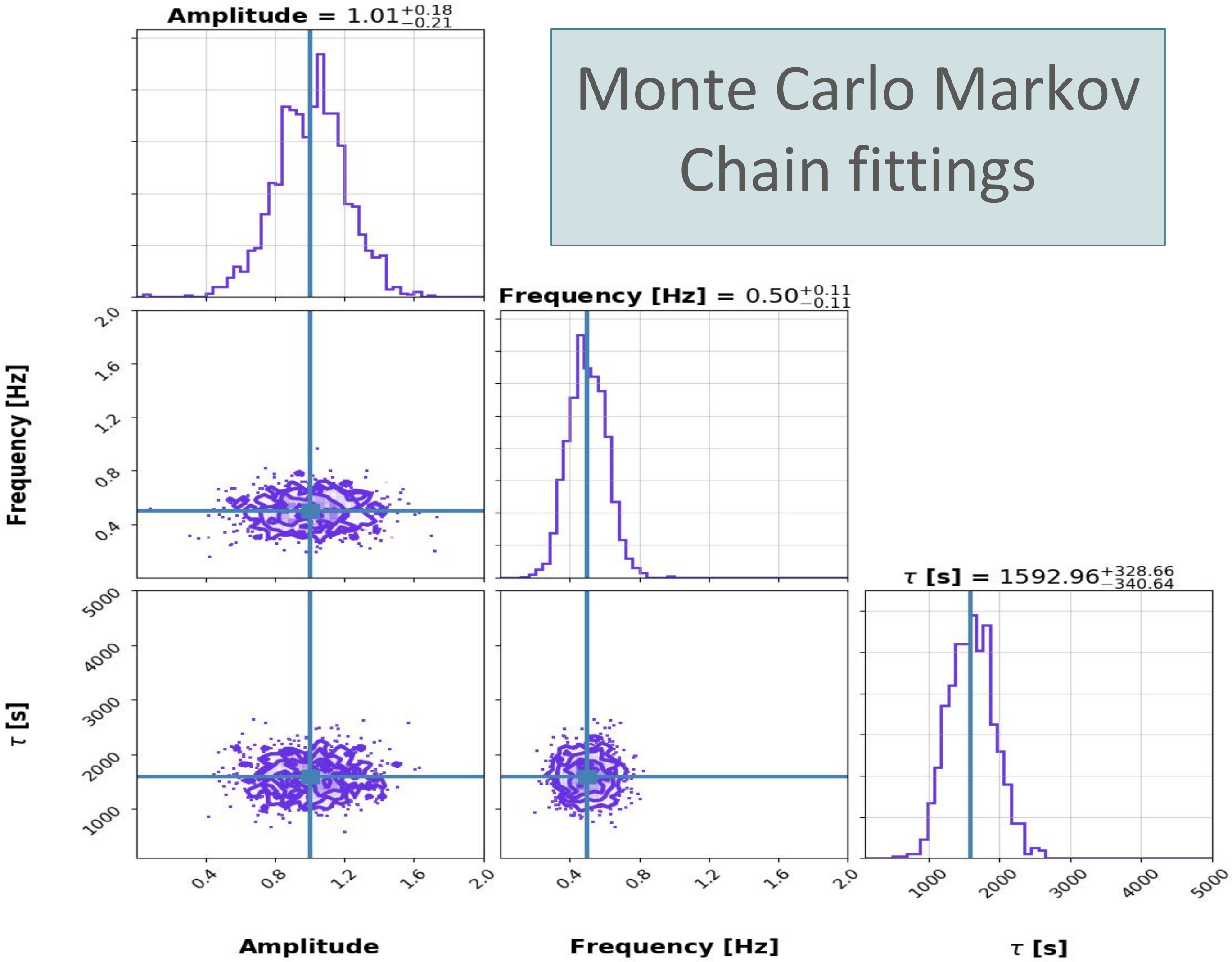


From the fit, τ is calculated as 3.02.

$$Q = \pi\tau f_{ij}$$

τ = 3.02
Q = $2.1*10^{4}$
ϕ = $4.8*10^{-5}$
σ^2 = $1.91 *10^{-5}$

Monte Carlo Markov Chain fittings



An alternative method to fitting a decaying sine wave to calculate τ

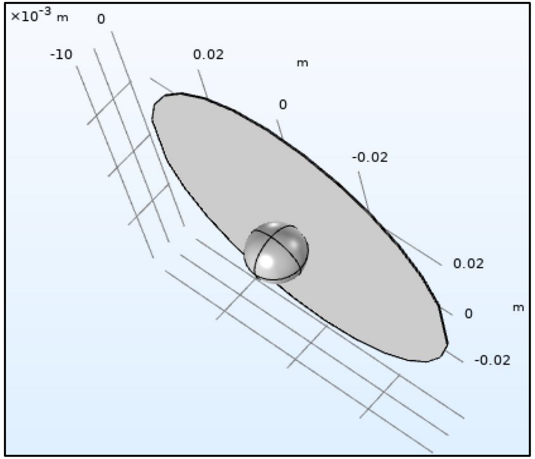
Loss Tomography

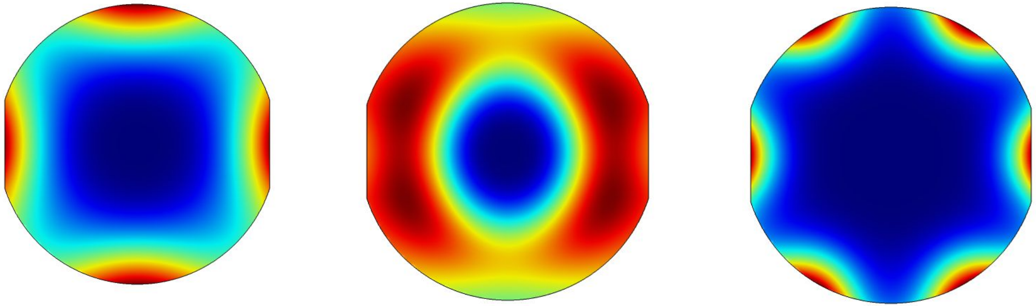
$$\begin{pmatrix} SER_{substrate} & SER_{coat} \\ SER_{substrate} & SER_{coat} \end{pmatrix}^{-1} \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} = \begin{pmatrix} \phi_{TE} \\ \phi_{Ak} \end{pmatrix}$$

COMSOL	EXPERIMENTAL	ANALYTIC
Strain energy in 5 µm coating on the disk/ total strain energy in disk as well as the substrate SER/ total strain energy - computed through finite element analysis software COMSOL	Total loss angle Variance: σ^2_{fit}	Loss angle contribution from thermoelastic loss in the disk coating.

COMSOL FEA - Eigenmodes

- Disk and point contact modeled in COMSOL
- Variance of experimental result of eigenfrequency and COMSOL result minimized by varying parameters (disk thickness and Young's modulus) in order to accurately represent the physical situation

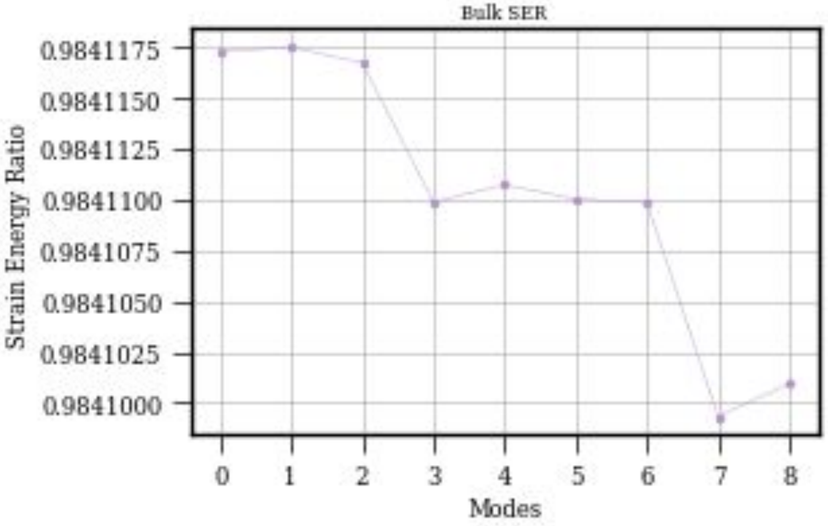


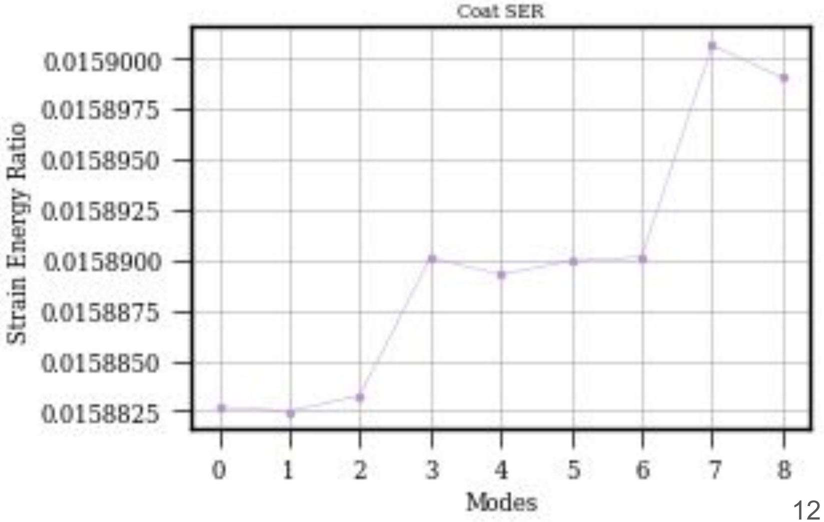


Strain energy density of different modes, clear that some modes have higher strain in the center or edge

COMSOL FEA - SER

- COMSOL to calculate the strain energy ratio (SER) for areas like bulk, surface, coat etc.
- Might want to select a mode with a lower SER in the bulk since it outweighs the coat





Loss Tomography- results

First, a simple example of solving for the surface and bulk losses using a 2 by 2 system

$$\begin{pmatrix} SER_{bulk1} & SER_{surf1} \\ SER_{bulk2} & SER_{surf2} \end{pmatrix}^{-1} \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} = \begin{pmatrix} \phi_{bulk} \\ \phi_{surf} \end{pmatrix}$$

Q factor	Loss angle	SER_{BULK}	$SER_{SURFACE}$
$5*10^6$	$0.2*10^{-6}$	0.984108	0.015892
$4*10^6$	$0.25*10^{-6}$	0.984117	0.015883

$$\begin{pmatrix} \phi_{sub} \\ \phi_{surf} \end{pmatrix} = \begin{matrix} 1.03 * 10^{-7} \\ 9.79 * 10^{-6} \end{matrix}$$

LIGO

Next Steps

- After completing the simple examples of loss tomography, the next step is to keep adding parameters
- For example, solve for the thermoelastic loss and use this to solve for the temperature of the disk and so on

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \begin{pmatrix} 1 & SER_{sub1} & SER_{surf1} \\ 1 & SER_{sub2} & SER_{surf2} \\ 1 & SER_{sub3} & SER_{surf3} \end{pmatrix}^{-1} = \begin{pmatrix} \phi_{TE} \\ \phi_{sub} \\ \phi_{surf} \end{pmatrix}$$

$$\phi_{th}(\omega) = \frac{Y\alpha^2 T}{\rho C_v} \frac{\omega\tau_{th}}{1+\omega^2\tau_{th}^2}$$

- After the first stage of this analysis is complete, we will use MCMC methods to make these same parameter estimations, and move on to measuring the loss of various thin film coatings

Acknowledgements

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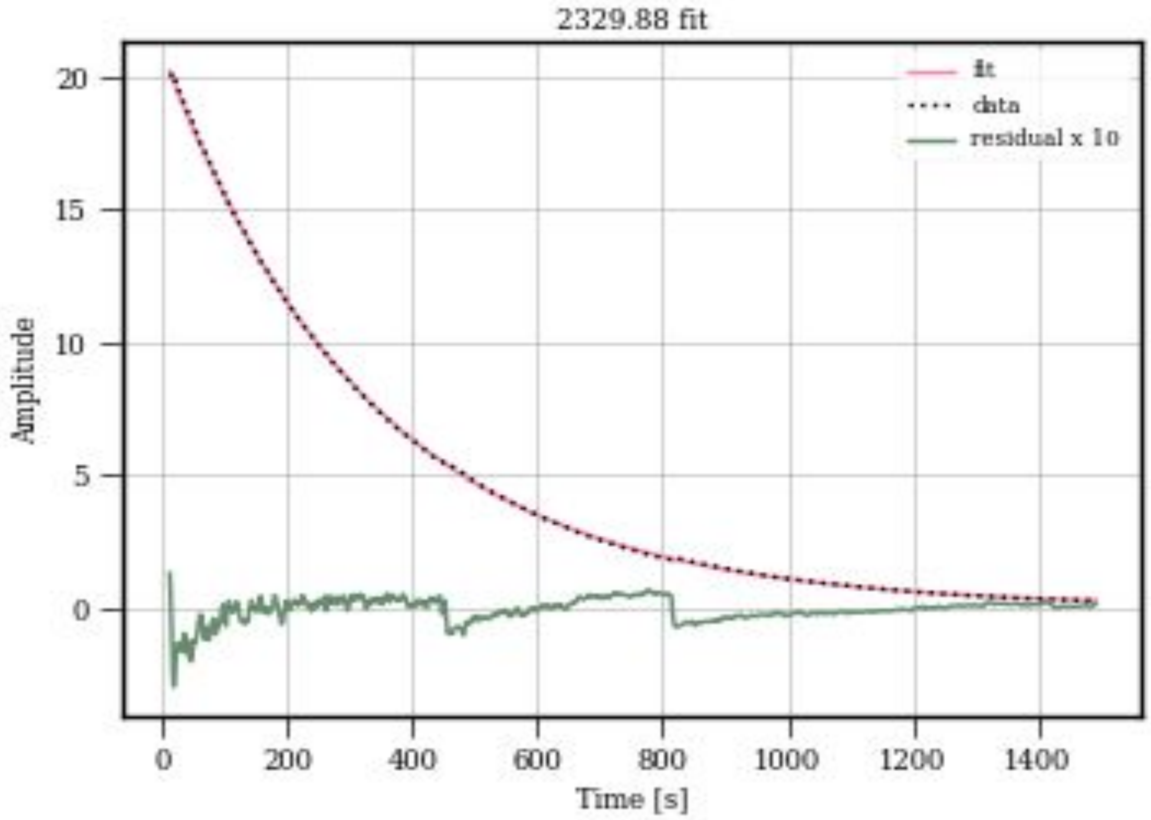
Amplitude locked loop

$$f(t) = Ae^{t/\tau} + C$$

- Excites a mode and allows ϕ, f, temperature, amplitude, and Q factor to be measured live.
- Can analyze amplitude data and *quickly* calculate Q factor



τ = 1876.39
Q= $2.4*10^6$
ϕ= $4.17*10^{-7}$
σ^2= $1.47*10^{-6}$

This mode was excited at around 180 K instead of room temp.

Drastic increase in Q

Loss Tomography

Combining the experimental, modelled, and analytic results the contributions of different loss sources can be found, providing insight on which modes should be excited to minimize certain losses

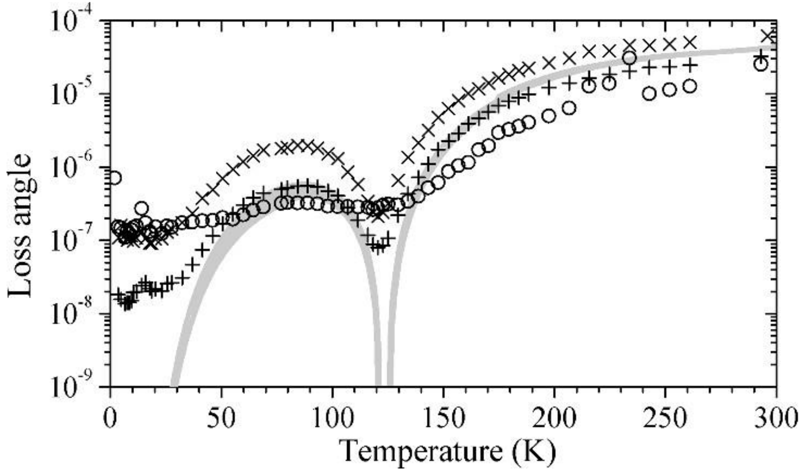
$$\phi_{970} = SER_{bulk,1}\phi_{1,bulk} + SER_{coat,1}\phi_{1,coat}$$
$$\phi_{1027} = SER_{bulk,2}\phi_{2,bulk} + SER_{coat,2}\phi_{2,coat}$$

$$\begin{pmatrix} \phi_{970} \\ \phi_{1027} \end{pmatrix} \begin{pmatrix} SER_{bulk,1} & SER_{coat,1} \\ SER_{bulk,2} & SER_{coat,2} \end{pmatrix}^{-1} = \begin{pmatrix} \phi_{1,bulk} & \phi_{1,coat} \\ \phi_{2,bulk} & \phi_{2,coat} \end{pmatrix}$$

RESULT

Final Loss Tomography equations

Thermoelastic loss



$$\phi_{th}(\omega) = \frac{Y\alpha^2 T}{\rho C_v} \frac{\omega\tau_{th}}{1+\omega^2\tau_{th}^2}$$

Akhiezer loss

$$\phi_{Ak}(\omega) = \frac{\Gamma_a^2 T C_v}{\rho c^2}\omega\tau_{Ak}$$