LIGO-G2000218.

Stacked-Triplet Ternary HR Coatings
Another Multimaterial Design Option

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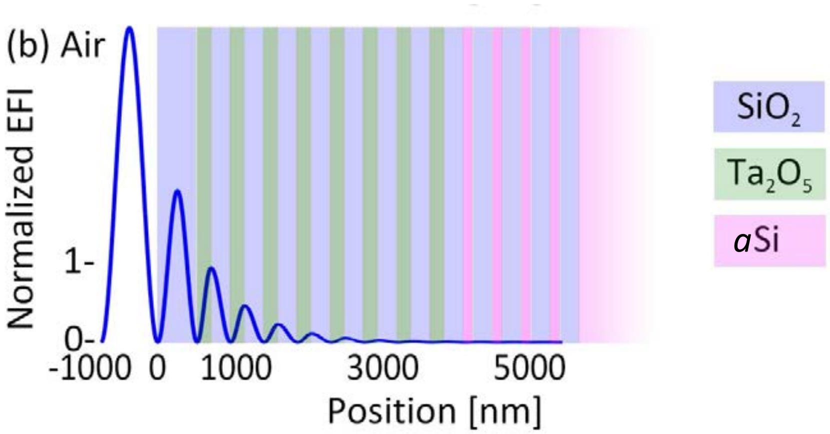
Virgo-LSC OWG Telecon, March 3rd 2020

Multi-Material HR Coatings

- Multimaterial coatings for GW detectors were introduced in:

  Steinlechner et al., PRD 91 (2015) 042001
  Yam et al., PRD 91 (2015) 042002

in the perspective of using an optically dense(r) but lossy material featuring low(er) mechanical losses compared to mainstream materials (Silica and Ti::Tantala) in the "deep" layers of the coating, where the field is suitably weak.



- The concept has been subsequently elaborated following several directions [Craig et al., PRL 122 (2019) 231102].

- The idea of using more than two materials is gaining momentum, in view of the development of new improved coating materials, including *a*Si [Birney et al. PRL 121 (2018) 191101], SiOx [Gras et al., LIGO-G1901619], Silicon Nitrides [Chao et al., LIGO-G192341], GeO_2 [Vajente et al., OWG] .

- In this communication we explore an alternative multimaterial coating design option based on *stacked (Bragg) triplets* (or more generally m-tuplets).

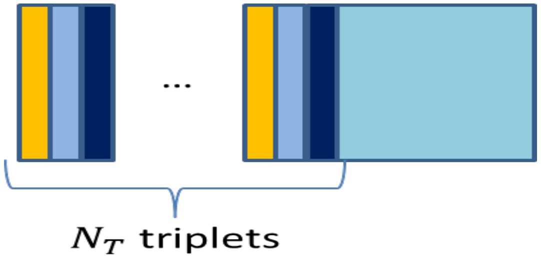
Larruquert Rule

For building HR stacked multiplet coatings using lossy materials :

> "The materials (in each m-tuplet) should be chosen so as to move *clockwise* in the complex refraction-index plane when going from the top to the bottom layer"

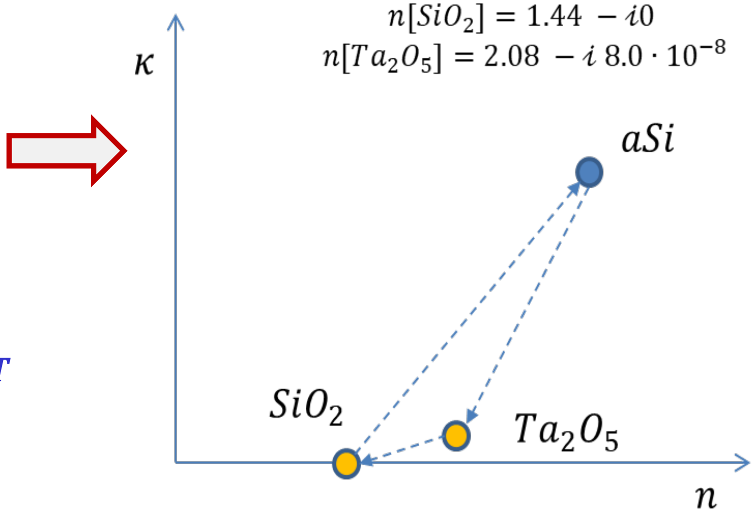
[J. Larruquert, JOSA A18 (2001) 2617, J. Larruquert, JOSA A21 (2004) 1750]

Consider, e.g., stacked-*triplet* coatings, using Silica, Tantala and *a*-Silicon.



$$n[aSi] = 3.43 - i1.2 \cdot 10^{-5}$$
$$n[SiO_2] = 1.44 - i0$$
$$n[Ta_2O_5] = 2.08 - i\,8.0 \cdot 10^{-8}$$

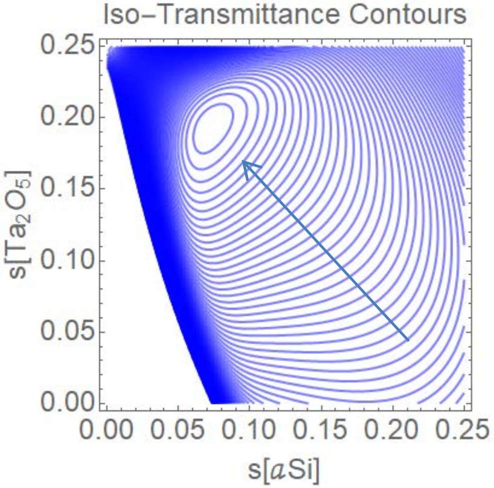
@10K
@1550nm

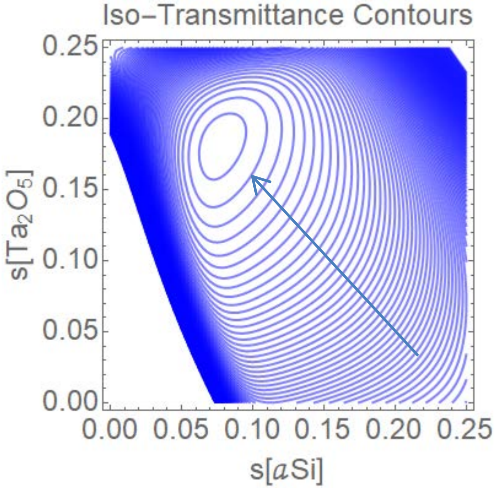


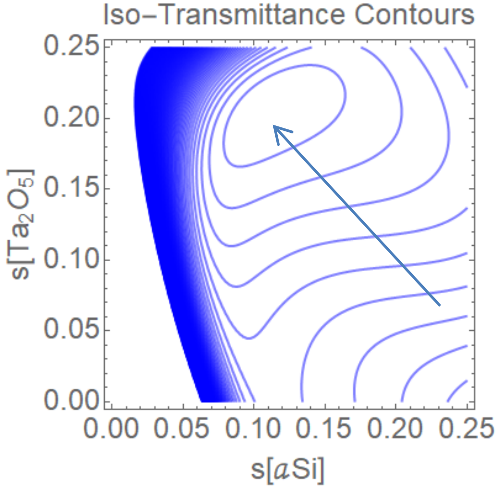
- **Let the optical thicknesses $s[aSi]$, and $s[Ta_2O_5]$ and the number N_T of triplets the design parameters;**
- **Assume all triplets as HWL. Hence $s[SiO_2] = 0.5 - (s[aSi] + s[Ta_2O_5])$.**

Larruquert Rule Illustrated . 15-Triplets Coating Transmittance

“Clockwise sequences”

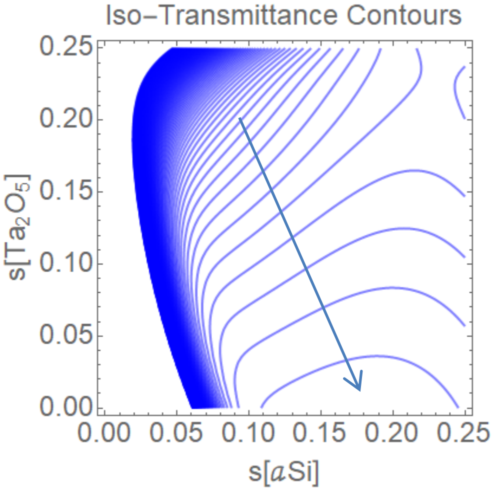

Silicon->Tantala->Silica

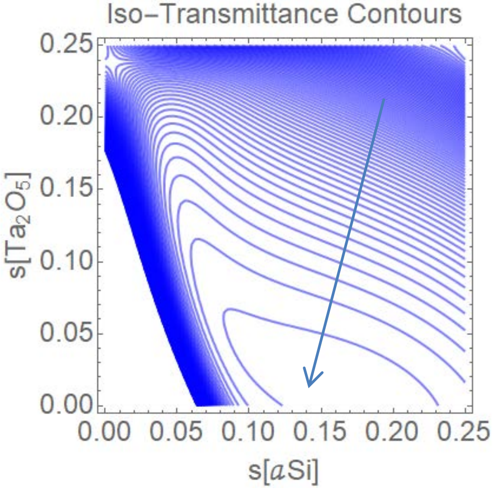

Tantala->Silica->Silicon

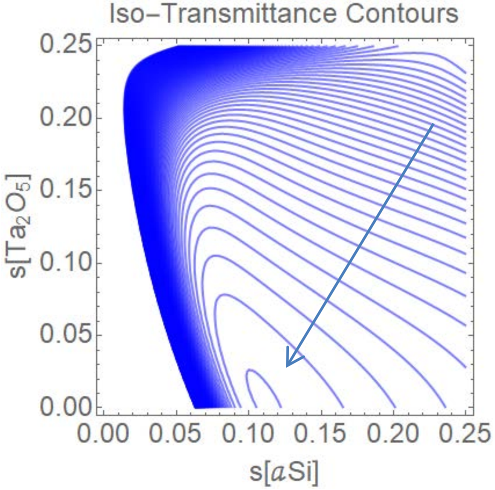

Silica->Silicon->Tantala

(arrow in plots points toward lower transmittance values)

“Countervlockwise sequences”


Silicon->Silica->Tantala

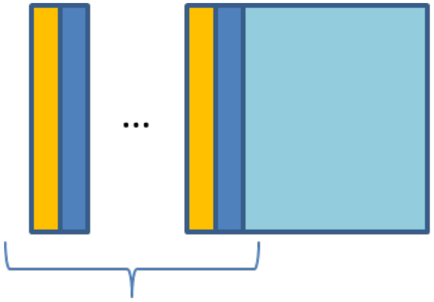

Tantala->Silicon->Silica


Silica->Tantala->Silicon

Stacked-Triplet HR Coating Design Test-Run

Design goals (ET-LF like) : $\tau_P = 5\,ppm\,,\ \pi_{max} = 5\,ppm$

Reference design: HWL-doublets
Tantala(🟨)/Silica(🟦)
QWL-layers
(with HWL-thick SiO_2 cap added on top)



17 doublets , $\tau_P = 4.558\,ppm$

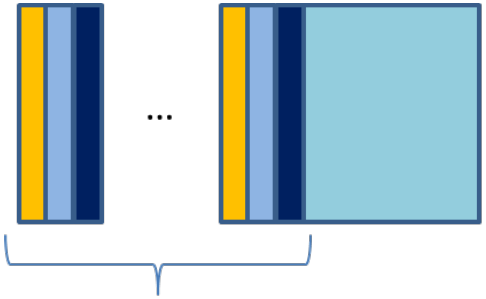
$\bar{\phi}_C = 15.948$

$\pi_L = 0.22\,ppm$

Material Parameters:

	n	k	Y [GPa]	ϕ @10K
aSi	3.48	$(1.2 \pm 0.2)\cdot 10^{-5}$@1550nm $(1.7 \pm 0.1)\cdot 10^{-4}$@1064nm	147	$1.7\cdot 10^{-5}$
Ti::Ta_2O_5	2.05	$0.008\cdot 10^{-5}$	140	$7.8\cdot 10^{-4}$
SiO_2	1.44	0	72	$8.5\cdot 10^{-4}$

Stacked-triplets design: HWL-triplets
Tantala(🟨)/Silica(🟦)/Silicon(⬛)
(or cyclic permutations thereof)
non-QWL layers
(with HWL-thick SiO_2 cap added on top)



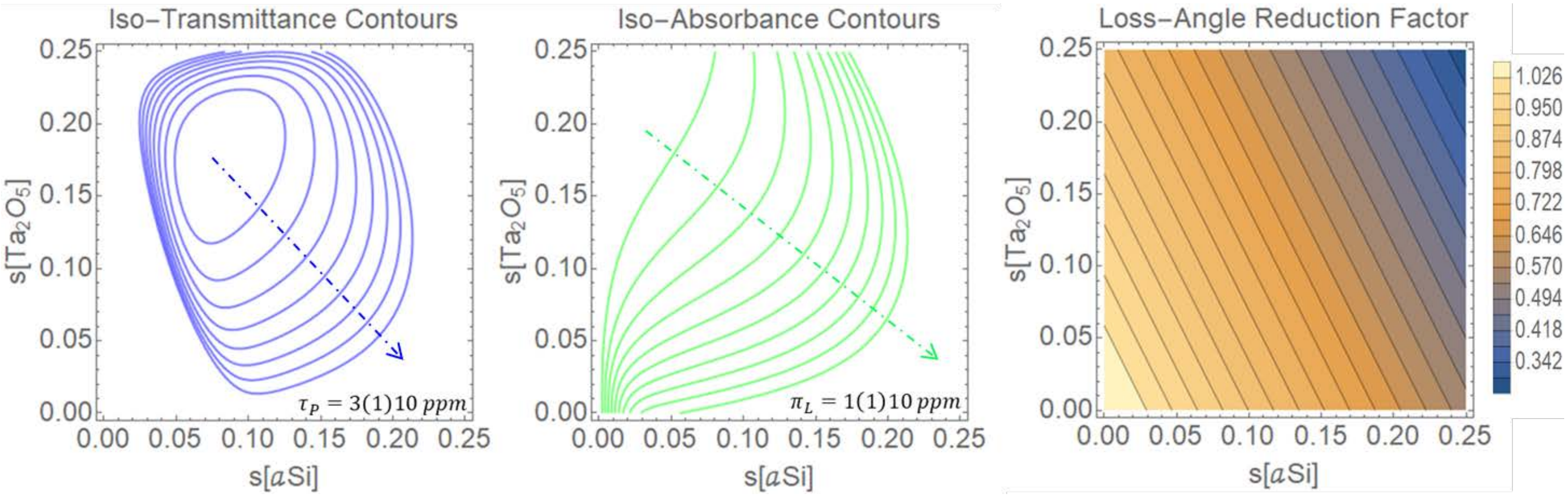
N_T triplets, $\tau_P = 5\,ppm$

$\bar{\phi}_C = ?$

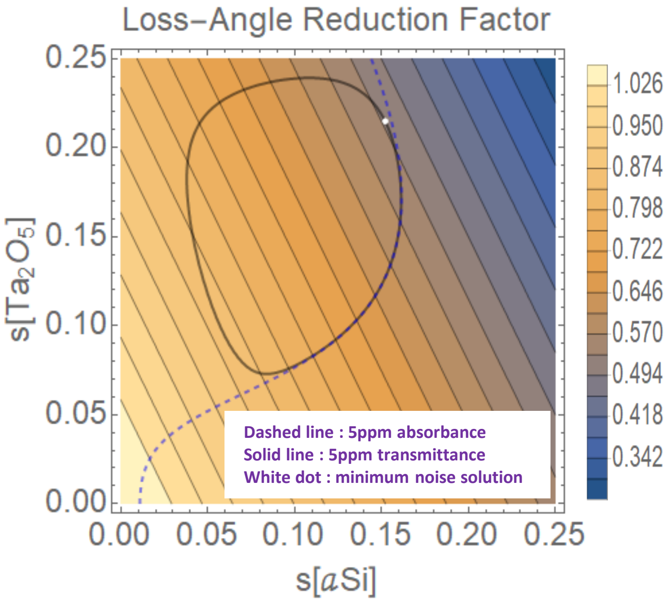
$\pi_L \leq 5\,ppm$

Stacked-Triplet Design – Fixed N_T , Fixed Material Sequence

Sequence: Tantala, Silica, Silicon; $N_T = 15$



→ Mininimum-noise design



Dashed line : 5ppm absorbance
Solid line : 5ppm transmittance
White dot : minimum noise solution

Stacked-Triplet Design Algorithm (prescribed τ_P and π_{max})

```
For all admissible (clockwise in complex index plane) Larruquert material sequences,
    While $N_T < N_{ref}$ ,
        • draw the $\tau_P = \tau_P^{(ref)}$ contour $\Theta$ in the domain $\mathcal{D}$
          $0 \leq s[aSi] \leq 0.25 \times 0 \leq s[Ta_2O_5] \leq 0.25$ ;
        • in the same domain draw the $\pi_L = \pi_{max}$ contour bounding the admissible region $\Lambda$ where $\pi_L < \pi_{max}$;
        • draw the constant $-\phi_C$ contours in $\mathcal{D}$;
        • identify the $(s[aSi], s[Ta_2O_5])$ point on $\Theta \cap \Lambda$
          for which $R_\phi$ is minimum;
    End while;
    Select the $N_T$ for which $R_\phi$ is minimum;
End for;
Select the sequence for which $R_\phi$ is minimum.
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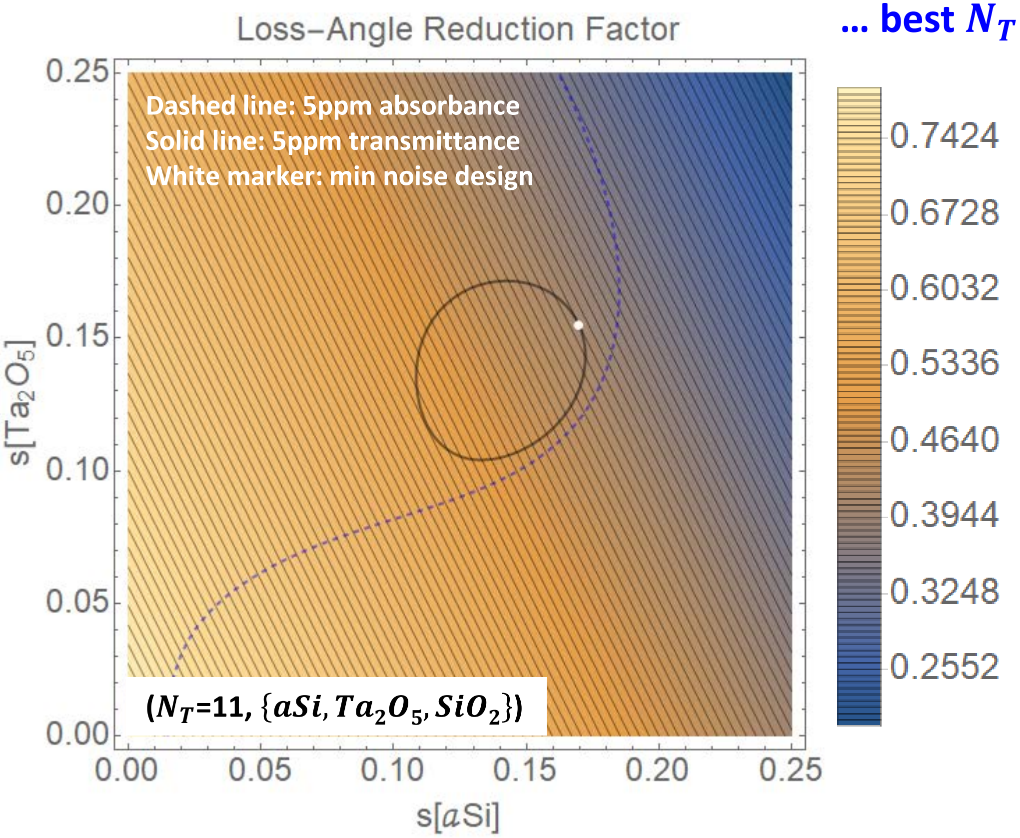
see previous slide

Optimum Stacked Triplet Design with $\tau_P = 5ppm$, $\pi_{max} = 5ppm$

Noise (coating loss-angle) reduction factor R_ϕ (smaller is better)

N_T mater. seq.	10	11	12	13	14	15	16	17
{aSi,Ti::Ta_2O_5,SiO_2}	no sol.	$\boxed{.406}$	.413	…	…	…	…	…
{Ti::Ta_2O_5,SiO_2,aSi}	no sol.	no sol.	.469	.482	…	…	…	…
{SiO_2,aSi,Ti::Ta_2O_5}	no sol.	no sol.	no sol.	no sol.	no sol.	no sol.	.584	.591



… best N_T = smallest N_T for which target τ_P is attained

Best stacked-triplet design:

sequence: Silicon, Ti::Tantala, Silica,

N_T = 11, { s[aSi], s[Ta2O5], s[SiO2] } =

= { 0.1698, 0.1550, 0.1752 }

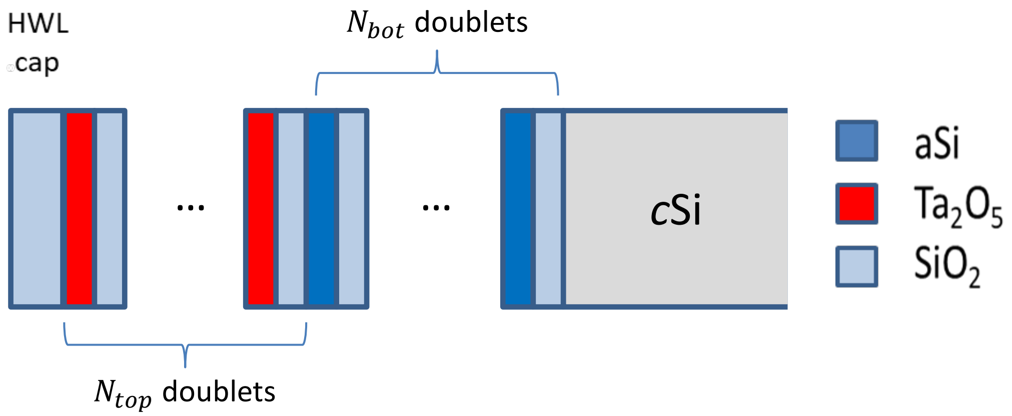
$\tau_P = 5\ ppm$; $\pi_L = 4.65\ ppm$;

$R_\phi = 0.406$

…how does this compare to best SY-ternary coating ?..

SY Ternary-Coatings in a Nutshell

First proposed/analysed in
- Steinlechner et al., PRD 91 (2015) 042001
- Yam et al., PRD 91 (2015) 042002



$$L_{HWL}(H_{QWL}\ L_{QWL})^{N_{top}}(H'_{QWL}\ L_{QWL})^{N_{bot}}$$

➡ Confirmed to be the *optimal design* (lowest thermal noise for $\tau_P \leq \tau_{ref}$ under a prescribed maximum absorbance constraint) *among all ternary (aSi, Ta_2O_5, SiO_2) multilayers consisting of QWL-thick layers,* by exhaustive search [Pierro et al., 2020] .

SY-Coatings References

- Steinlechner et al., PRD 91 (2015) 042001 (first multimaterial (MM) design discussion)
- Yam et al., PRD 91 (2015) 042002 (multimaterial modeling discussion)
- Steinlechner and Martin, PRD 93 (2016) 102001 (discusses design, including cSi top layer)
- Steinlechner et al. PRD 96 (2017) 022007 (discusses Si_3N_4 as candidate MM ingredient)
- Steinlechner et al., Phil. Trans. A376 (2018) 20170282 (nice review of subject)
- Pan et al., PRD 98 (2018) 102001 (discusses $SiN_{0.4}H_{0.79}$ as candidate MM ingredient)
- Byrney et al., PRL 121 (2018) 191101 (best-ever aSi obtained and discussed)
- Craig et al., PRL 122 (2019) 231102 ($Si::HfO_2$ as candidate MM L-index 3G-cryo material)
- Tait et al., LIGO-P1900002 (measurements on first MM prototype)

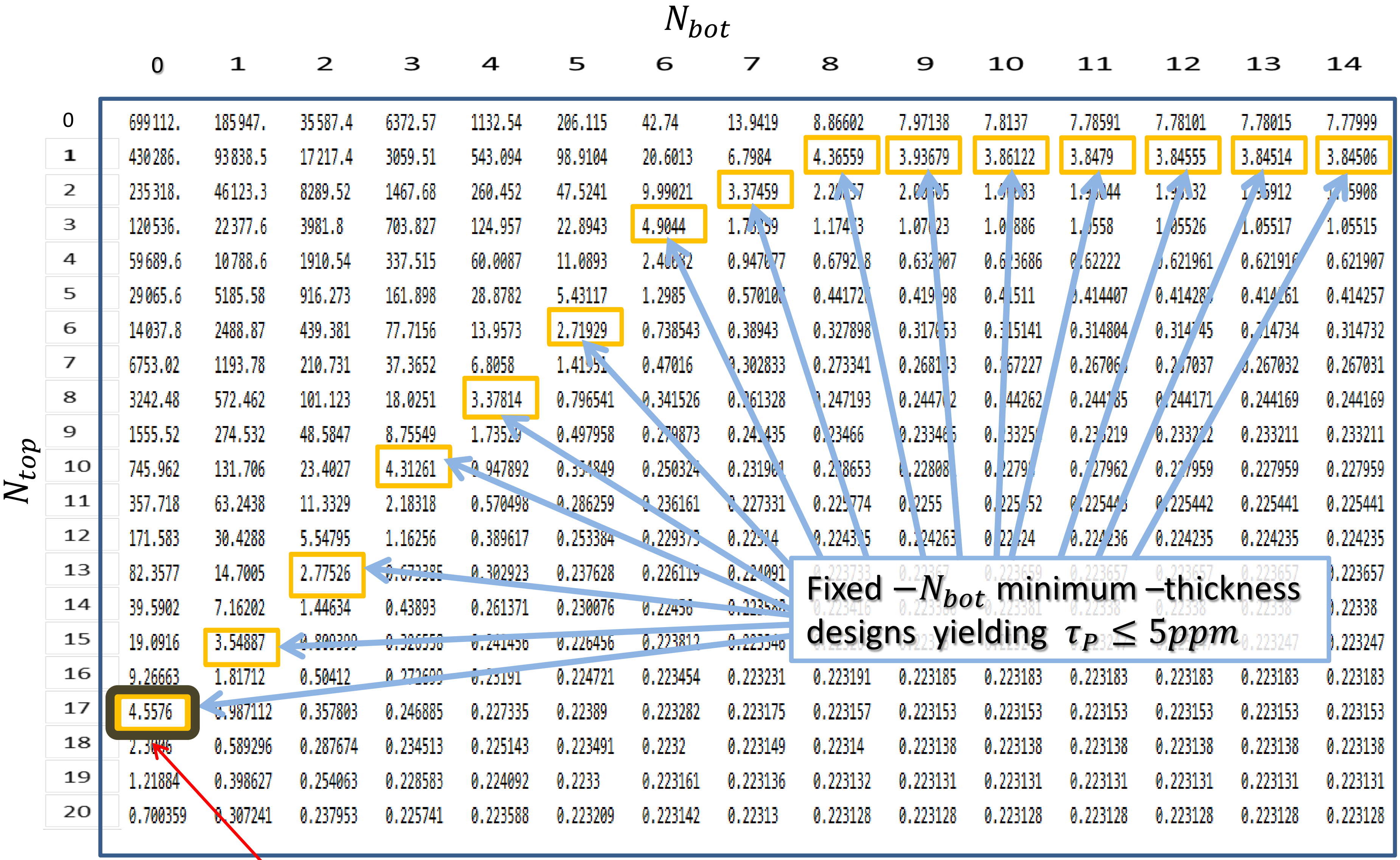
SY-Coatings Transmittance [ppm] vs. (N_{top}, N_{bot})

N_{bot}

N_{top}

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	699112.	185947.	35587.4	6372.57	1132.54	206.115	42.74	13.9419	8.86602	7.97138	7.8137	7.78591	7.78101	7.78015	7.77999
1	430286.	93838.5	17217.4	3059.51	543.094	98.9104	20.6013	6.7984	4.36559	3.93679	3.86122	3.8479	3.84555	3.84514	3.84506
2	235318.	46123.3	8289.52	1467.68	260.452	47.5241	9.99021	3.37459	2.20857	2.00305	1.96683	1.96044	1.95932	1.95912	1.95908
3	120536.	22377.6	3981.8	703.827	124.957	22.8943	4.9044	1.73359	1.17473	1.07623	1.05886	1.0558	1.05526	1.05517	1.05515
4	59689.6	10788.6	1910.54	337.515	60.0087	11.0893	2.46682	0.947077	0.679218	0.632007	0.623686	0.62222	0.621961	0.621916	0.621907
5	29065.6	5185.58	916.273	161.898	28.8782	5.43117	1.2985	0.570108	0.441726	0.419098	0.41511	0.414407	0.414283	0.414261	0.414257
6	14037.8	2488.87	439.381	77.7156	13.9573	2.71929	0.738543	0.38943	0.327898	0.317053	0.315141	0.314804	0.314745	0.314734	0.314732
7	6753.02	1193.78	210.731	37.3652	6.8058	1.41951	0.47016	0.302833	0.273341	0.268143	0.267227	0.267066	0.267037	0.267032	0.267031
8	3242.48	572.462	101.123	18.0251	3.37814	0.796541	0.341526	0.261328	0.247193	0.244702	0.244262	0.244185	0.244171	0.244169	0.244169
9	1555.52	274.532	48.5847	8.75549	1.73529	0.497958	0.279873	0.241435	0.23466	0.233466	0.233256	0.233219	0.233212	0.233211	0.233211
10	745.962	131.706	23.4027	4.31261	0.947892	0.354849	0.250324	0.231901	0.228653	0.228081	0.22798	0.227962	0.227959	0.227959	0.227959
11	357.718	63.2438	11.3329	2.18318	0.570498	0.286259	0.236161	0.227331	0.225774	0.2255	0.225452	0.225443	0.225442	0.225441	0.225441
12	171.583	30.4288	5.54795	1.16256	0.389617	0.253384	0.229373	0.22514	0.224395	0.224263	0.22424	0.224236	0.224235	0.224235	0.224235
13	82.3577	14.7005	2.77526	0.673385	0.302923	0.237628	0.226119	0.224091	0.223733	0.22367	0.223659	0.223657	0.223657	0.223657	0.223657
14	39.5902	7.16202	1.44634	0.43893	0.261371	0.230076	0.22456	0.223588	0.223416	0.223386	0.223381	0.22338	0.22338	0.22338	0.22338
15	19.0916	3.54887	0.809399	0.326558	0.241456	0.226456	0.223812	0.223346	0.223264	0.22325	0.223247	0.223247	0.223247	0.223247	0.223247
16	9.26663	1.81712	0.50412	0.272699	0.23191	0.224721	0.223454	0.223231	0.223191	0.223185	0.223183	0.223183	0.223183	0.223183	0.223183
17	4.5576	0.987112	0.357803	0.246885	0.227335	0.22389	0.223282	0.223175	0.223157	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153
18	2.3006	0.589296	0.287674	0.234513	0.225143	0.223491	0.2232	0.223149	0.22314	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138
19	1.21884	0.398627	0.254063	0.228583	0.224092	0.2233	0.223161	0.223136	0.223132	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131
20	0.700359	0.307241	0.237953	0.225741	0.223588	0.223209	0.223142	0.22313	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128

SY-Coatings Transmittance [ppm] vs. (N_{top}, N_{bot})



N_{bot}

N_{top}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	699112.	185947.	35587.4	6372.57	1132.54	206.115	42.74	13.9419	8.86602	7.97138	7.8137	7.78591	7.78101	7.78015	7.77999
1	430286.	93838.5	17217.4	3059.51	543.094	98.9104	20.6013	6.7984	4.36559	3.93679	3.86122	3.8479	3.84555	3.84514	3.84506
2	235318.	46123.3	8289.52	1467.68	260.452	47.5241	9.99021	3.37459	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	120536.	22377.6	3981.8	703.827	124.957	22.8943	4.9044	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.05517	1.05515
4	59689.6	10788.6	1910.54	337.515	60.0087	11.0893	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.62222	0.621961	[illegible]	0.621907
5	29065.6	5185.58	916.273	161.898	28.8782	5.43117	1.2985	[illegible]	[illegible]	[illegible]	[illegible]	0.414407	[illegible]	[illegible]	0.414257
6	14037.8	2488.87	439.381	77.7156	13.9573	2.71929	0.738543	0.38943	0.327898	[illegible]	[illegible]	0.314804	[illegible]	[illegible]	0.314732
7	6753.02	1193.78	210.731	37.3652	6.8058	[illegible]	0.47016	0.302833	0.273341	[illegible]	[illegible]	[illegible]	[illegible]	0.267032	0.267031
8	3242.48	572.462	101.123	18.0251	3.37814	0.796541	0.341526	[illegible]	0.247193	[illegible]	[illegible]	[illegible]	0.244171	0.244169	0.244169
9	1555.52	274.532	48.5847	8.75549	[illegible]	0.497958	[illegible]	[illegible]	0.23466	[illegible]	[illegible]	[illegible]	[illegible]	0.233211	0.233211
10	745.962	131.706	23.4027	4.31261	0.947892	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.227959	0.227959
11	357.718	63.2438	11.3329	2.18318	0.570498	0.286259	0.236161	0.227331	[illegible]	[illegible]	[illegible]	[illegible]	0.225442	0.225441	0.225441
12	171.583	30.4288	5.54795	1.16256	0.389617	0.253384	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.224235	0.224235	0.224235
13	82.3577	14.7005	2.77526	[illegible]	0.302923	0.237628	[illegible]	[illegible]	0.223733	0.22367	0.223659	0.223657	0.223657	0.223657	0.223657
14	39.5902	7.16202	1.44634	0.43893	0.261371	0.230076	[illegible]	[illegible]	0.223416	[illegible]	0.223381	0.22338	0.22338	0.22338	0.22338
15	19.0916	3.54887	[illegible]	[illegible]	0.241456	0.226456	0.223812	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.223247	0.223247
16	9.26663	1.81712	0.50412	[illegible]	0.23191	0.224721	0.223454	0.223231	0.223191	0.223185	0.223183	0.223183	0.223183	0.223183	0.223183
17	4.5576	0.987112	0.357803	0.246885	0.227335	0.22389	0.223282	0.223175	0.223157	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153
18	[illegible]	0.589296	0.287674	0.234513	0.225143	0.223491	0.2232	0.223149	0.22314	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138
19	1.21884	0.398627	0.254063	0.228583	0.224092	0.2233	0.223161	0.223136	0.223132	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131
20	0.700359	0.307241	0.237953	0.225741	0.223588	0.223209	0.223142	0.22313	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128

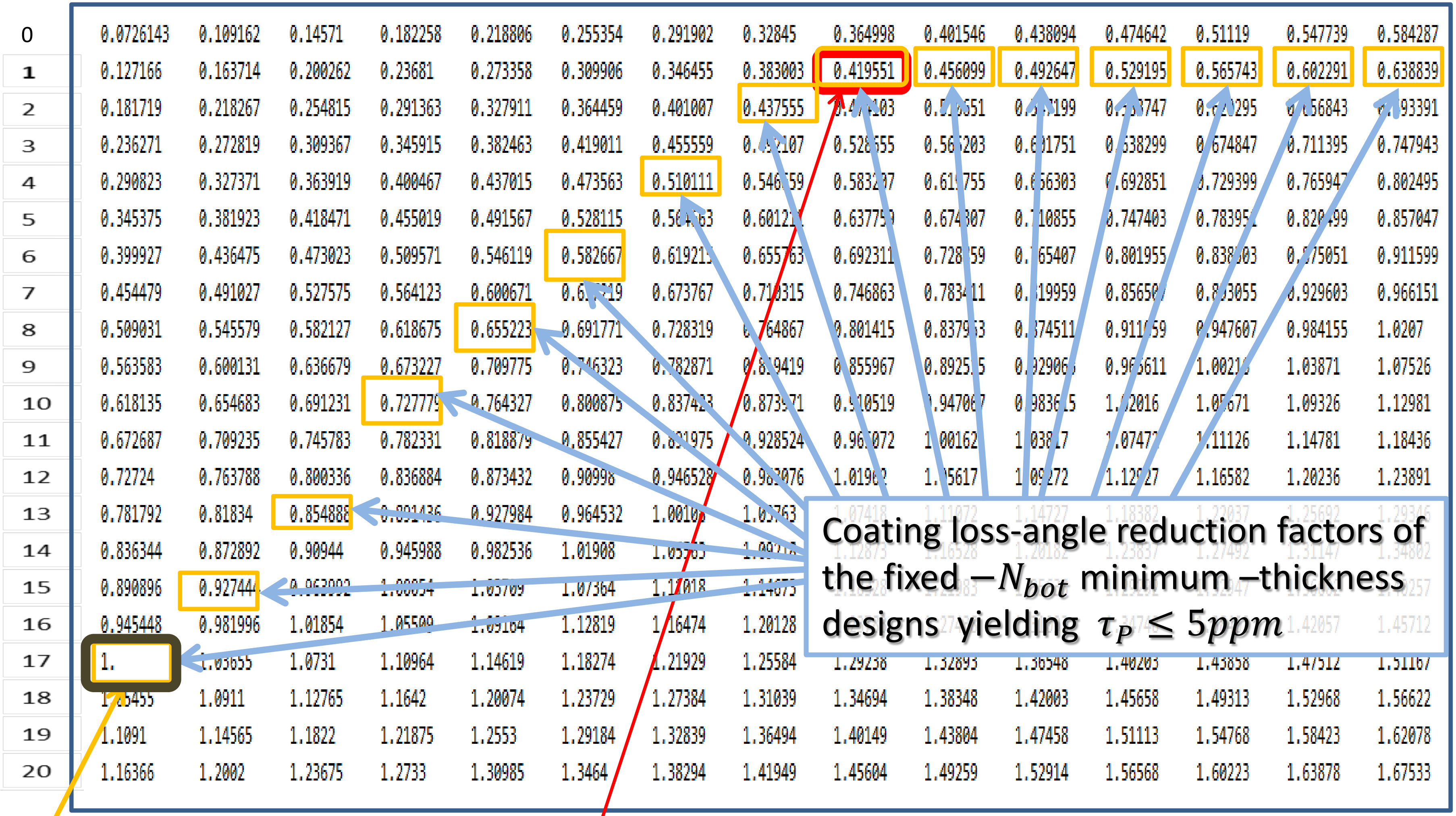
Fixed $-N_{bot}$ minimum –thickness designs yielding $\tau_P \leq 5ppm$

Reference Silica/Ti::Tantala-only SD design with $\tau_P \leq 5ppm$@1550nm

SY-Coatings ϕ_C Reduction Factor vs (N_{top}, N_{bot})

	N_{bot}														
N_{top}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.0726143	0.109162	0.14571	0.182258	0.218806	0.255354	0.291902	0.32845	0.364998	0.401546	0.438094	0.474642	0.51119	0.547739	0.584287
1	0.127166	0.163714	0.200262	0.23681	0.273358	0.309906	0.346455	0.383003	0.419551	0.456099	0.492647	0.529195	0.565743	0.602291	0.638839
2	0.181719	0.218267	0.254815	0.291363	0.327911	0.364459	0.401007	0.437555	0.474103	0.510651	0.547199	0.583747	0.620295	0.656843	0.693391
3	0.236271	0.272819	0.309367	0.345915	0.382463	0.419011	0.455559	0.492107	0.528655	0.565203	0.601751	0.638299	0.674847	0.711395	0.747943
4	0.290823	0.327371	0.363919	0.400467	0.437015	0.473563	0.510111	0.546659	0.583207	0.619755	0.656303	0.692851	0.729399	0.765947	0.802495
5	0.345375	0.381923	0.418471	0.455019	0.491567	0.528115	0.564663	0.601211	0.637759	0.674307	0.710855	0.747403	0.783951	0.820499	0.857047
6	0.399927	0.436475	0.473023	0.509571	0.546119	0.582667	0.619215	0.655763	0.692311	0.728859	0.765407	0.801955	0.838503	0.875051	0.911599
7	0.454479	0.491027	0.527575	0.564123	0.600671	0.637219	0.673767	0.710315	0.746863	0.783411	0.819959	0.856507	0.893055	0.929603	0.966151
8	0.509031	0.545579	0.582127	0.618675	0.655223	0.691771	0.728319	0.764867	0.801415	0.837963	0.874511	0.911059	0.947607	0.984155	1.0207
9	0.563583	0.600131	0.636679	0.673227	0.709775	0.746323	0.782871	0.819419	0.855967	0.892515	0.929063	0.965611	1.00216	1.03871	1.07526
10	0.618135	0.654683	0.691231	0.727779	0.764327	0.800875	0.837423	0.873971	0.910519	0.947067	0.983615	1.02016	1.05671	1.09326	1.12981
11	0.672687	0.709235	0.745783	0.782331	0.818879	0.855427	0.891975	0.928524	0.965072	1.00162	1.03817	1.07472	1.11126	1.14781	1.18436
12	0.72724	0.763788	0.800336	0.836884	0.873432	0.90998	0.946528	0.983076	1.01962	1.05617	1.09272	1.12927	1.16582	1.20236	1.23891
13	0.781792	0.81834	0.854888	0.891436	0.927984	0.964532	1.00108	1.03763	1.07418	1.11072	1.14727	1.18382	1.22037	1.25692	1.29346
14	0.836344	0.872892	0.90944	0.945988	0.982536	1.01908	1.05563	1.09218	1.12873	1.16528	1.20182	1.23837	1.27492	1.31147	1.34802
15	0.890896	0.927444	0.963992	1.00054	1.03709	1.07364	1.11018	1.14673	1.18328	1.21983	1.25638	1.29292	1.32947	1.36602	1.40257
16	0.945448	0.981996	1.01854	1.05509	1.09164	1.12819	1.16474	1.20128	1.23783	1.27438	1.31093	1.34748	1.38402	1.42057	1.45712
17	1.	1.03655	1.0731	1.10964	1.14619	1.18274	1.21929	1.25584	1.29238	1.32893	1.36548	1.40203	1.43858	1.47512	1.51167
18	1.05455	1.0911	1.12765	1.1642	1.20074	1.23729	1.27384	1.31039	1.34694	1.38348	1.42003	1.45658	1.49313	1.52968	1.56622
19	1.1091	1.14565	1.1822	1.21875	1.2553	1.29184	1.32839	1.36494	1.40149	1.43804	1.47458	1.51113	1.54768	1.58423	1.62078
20	1.16366	1.2002	1.23675	1.2733	1.30985	1.3464	1.38294	1.41949	1.45604	1.49259	1.52914	1.56568	1.60223	1.63878	1.67533

SY-Coatings ϕ_C Reduction Factor vs (N_{top}, N_{bot})



N_{bot}

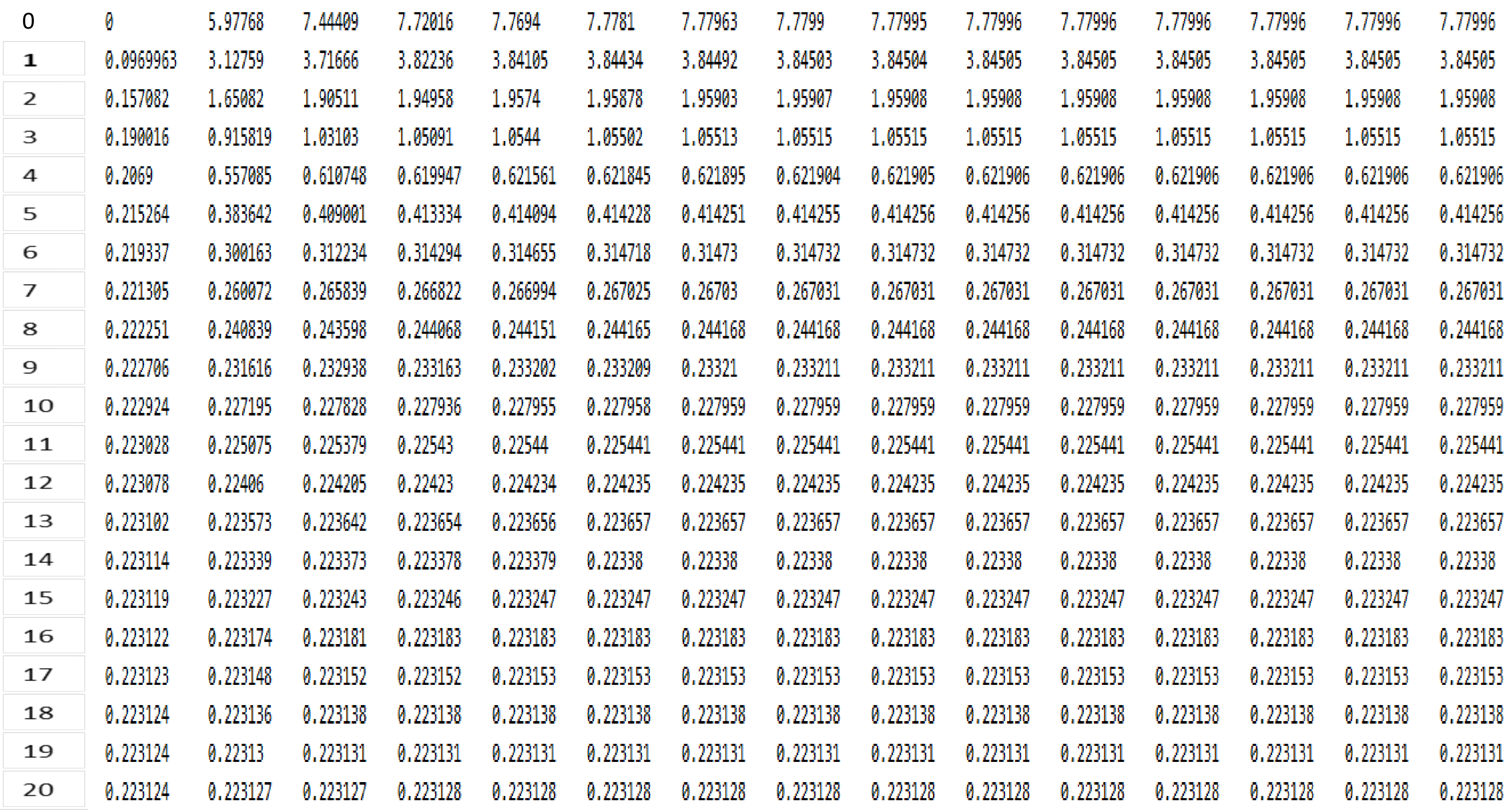
N_{top}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.0726143	0.109162	0.14571	0.182258	0.218806	0.255354	0.291902	0.32845	0.364998	0.401546	0.438094	0.474642	0.51119	0.547739	0.584287
1	0.127166	0.163714	0.200262	0.23681	0.273358	0.309906	0.346455	0.383003	0.419551	0.456099	0.492647	0.529195	0.565743	0.602291	0.638839
2	0.181719	0.218267	0.254815	0.291363	0.327911	0.364459	0.401007	0.437555	0.474103	0.510651	0.547199	0.583747	0.620295	0.656843	0.693391
3	0.236271	0.272819	0.309367	0.345915	0.382463	0.419011	0.455559	0.492107	0.528655	0.565203	0.601751	0.638299	0.674847	0.711395	0.747943
4	0.290823	0.327371	0.363919	0.400467	0.437015	0.473563	0.510111	0.546659	0.583207	0.619755	0.656303	0.692851	0.729399	0.765947	0.802495
5	0.345375	0.381923	0.418471	0.455019	0.491567	0.528115	0.564663	0.601211	0.637759	0.674307	0.710855	0.747403	0.783951	0.820499	0.857047
6	0.399927	0.436475	0.473023	0.509571	0.546119	0.582667	0.619215	0.655763	0.692311	0.728859	0.765407	0.801955	0.838503	0.875051	0.911599
7	0.454479	0.491027	0.527575	0.564123	0.600671	0.637219	0.673767	0.710315	0.746863	0.783411	0.819959	0.856507	0.893055	0.929603	0.966151
8	0.509031	0.545579	0.582127	0.618675	0.655223	0.691771	0.728319	0.764867	0.801415	0.837963	0.874511	0.911059	0.947607	0.984155	1.0207
9	0.563583	0.600131	0.636679	0.673227	0.709775	0.746323	0.782871	0.819419	0.855967	0.892515	0.929063	0.965611	1.00216	1.03871	1.07526
10	0.618135	0.654683	0.691231	0.727779	0.764327	0.800875	0.837423	0.873971	0.910519	0.947067	0.983615	1.02016	1.05671	1.09326	1.12981
11	0.672687	0.709235	0.745783	0.782331	0.818879	0.855427	0.891975	0.928524	0.965072	1.00162	1.03817	1.07472	1.11126	1.14781	1.18436
12	0.72724	0.763788	0.800336	0.836884	0.873432	0.90998	0.946528	0.983076	1.01962	1.05617	1.09272	1.12927	1.16582	1.20236	1.23891
13	0.781792	0.81834	0.854888	0.891436	0.927984	0.964532	1.00108	1.03763	1.07418	1.11072	1.14727	1.18382	1.22037	1.25692	1.29346
14	0.836344	0.872892	0.90944	0.945988	0.982536	1.01908	1.05563	1.09218	1.12873	1.16528	1.20182	1.23837	1.27492	1.31147	1.34802
15	0.890896	0.927444	0.963992	1.00054	1.03709	1.07364	1.11018	1.14673	1.18328	1.21983	[illegible]	1.29292	1.32947	[illegible]	1.40257
16	0.945448	0.981996	1.01854	1.05509	1.09164	1.12819	1.16474	1.20128	[illegible]	[illegible]	[illegible]	1.34748	[illegible]	1.42057	1.45712
17	1.	1.03655	1.0731	1.10964	1.14619	1.18274	1.21929	1.25584	1.29238	1.32893	1.36548	1.40203	1.43858	1.47512	1.51167
18	1.05455	1.0911	1.12765	1.1642	1.20074	1.23729	1.27384	1.31039	1.34694	1.38348	1.42003	1.45658	1.49313	1.52968	1.56622
19	1.1091	1.14565	1.1822	1.21875	1.2553	1.29184	1.32839	1.36494	1.40149	1.43804	1.47458	1.51113	1.54768	1.58423	1.62078
20	1.16366	1.2002	1.23675	1.2733	1.30985	1.3464	1.38294	1.41949	1.45604	1.49259	1.52914	1.56568	1.60223	1.63878	1.67533

Coating loss-angle reduction factors of the fixed $-N_{bot}$ minimum –thickness designs yielding $\tau_P \leq 5ppm$

Reference design

Best (lowest-noise) SY-design with $\tau_P \leq 5ppm$

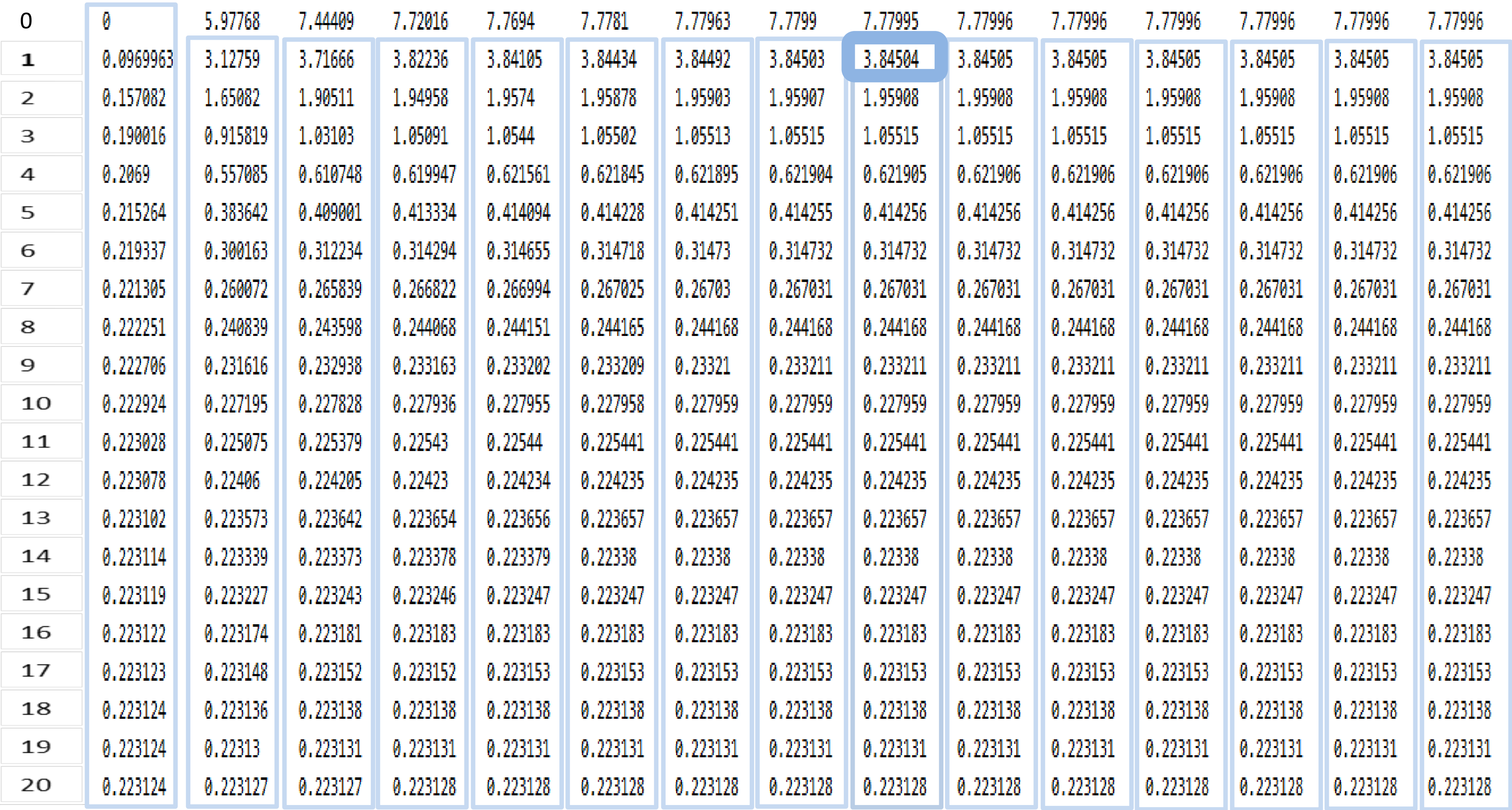
SY-Coatings Absorbance [ppm] vs. (N_{top}, N_{bot})



N_{bot}

N_{top}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	5.97768	7.44409	7.72016	7.7694	7.7781	7.77963	7.7799	7.77995	7.77996	7.77996	7.77996	7.77996	7.77996	7.77996
1	0.0969963	3.12759	3.71666	3.82236	3.84105	3.84434	3.84492	3.84503	3.84504	3.84505	3.84505	3.84505	3.84505	3.84505	3.84505
2	0.157082	1.65082	1.90511	1.94958	1.9574	1.95878	1.95903	1.95907	1.95908	1.95908	1.95908	1.95908	1.95908	1.95908	1.95908
3	0.190016	0.915819	1.03103	1.05091	1.0544	1.05502	1.05513	1.05515	1.05515	1.05515	1.05515	1.05515	1.05515	1.05515	1.05515
4	0.2069	0.557085	0.610748	0.619947	0.621561	0.621845	0.621895	0.621904	0.621905	0.621906	0.621906	0.621906	0.621906	0.621906	0.621906
5	0.215264	0.383642	0.409001	0.413334	0.414094	0.414228	0.414251	0.414255	0.414256	0.414256	0.414256	0.414256	0.414256	0.414256	0.414256
6	0.219337	0.300163	0.312234	0.314294	0.314655	0.314718	0.31473	0.314732	0.314732	0.314732	0.314732	0.314732	0.314732	0.314732	0.314732
7	0.221305	0.260072	0.265839	0.266822	0.266994	0.267025	0.26703	0.267031	0.267031	0.267031	0.267031	0.267031	0.267031	0.267031	0.267031
8	0.222251	0.240839	0.243598	0.244068	0.244151	0.244165	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168
9	0.222706	0.231616	0.232938	0.233163	0.233202	0.233209	0.23321	0.233211	0.233211	0.233211	0.233211	0.233211	0.233211	0.233211	0.233211
10	0.222924	0.227195	0.227828	0.227936	0.227955	0.227958	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959
11	0.223028	0.225075	0.225379	0.22543	0.22544	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441
12	0.223078	0.22406	0.224205	0.22423	0.224234	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235
13	0.223102	0.223573	0.223642	0.223654	0.223656	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657
14	0.223114	0.223339	0.223373	0.223378	0.223379	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338
15	0.223119	0.223227	0.223243	0.223246	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247
16	0.223122	0.223174	0.223181	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183
17	0.223123	0.223148	0.223152	0.223152	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153
18	0.223124	0.223136	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138
19	0.223124	0.22313	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131
20	0.223124	0.223127	0.223127	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128

SY-Coatings Absorbance [ppm] vs. (N_{top}, N_{bot})

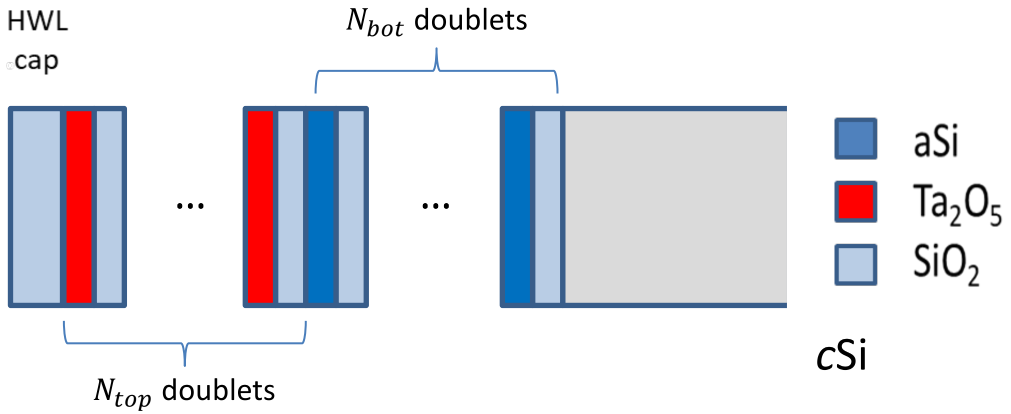


N_{bot}

N_{top}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	5.97768	7.44409	7.72016	7.7694	7.7781	7.77963	7.7799	7.77995	7.77996	7.77996	7.77996	7.77996	7.77996	7.77996
1	0.0969963	3.12759	3.71666	3.82236	3.84105	3.84434	3.84492	3.84503	3.84504	3.84505	3.84505	3.84505	3.84505	3.84505	3.84505
2	0.157082	1.65082	1.90511	1.94958	1.9574	1.95878	1.95903	1.95907	1.95908	1.95908	1.95908	1.95908	1.95908	1.95908	1.95908
3	0.190016	0.915819	1.03103	1.05091	1.0544	1.05502	1.05513	1.05515	1.05515	1.05515	1.05515	1.05515	1.05515	1.05515	1.05515
4	0.2069	0.557085	0.610748	0.619947	0.621561	0.621845	0.621895	0.621904	0.621905	0.621906	0.621906	0.621906	0.621906	0.621906	0.621906
5	0.215264	0.383642	0.409001	0.413334	0.414094	0.414228	0.414251	0.414255	0.414256	0.414256	0.414256	0.414256	0.414256	0.414256	0.414256
6	0.219337	0.300163	0.312234	0.314294	0.314655	0.314718	0.31473	0.314732	0.314732	0.314732	0.314732	0.314732	0.314732	0.314732	0.314732
7	0.221305	0.260072	0.265839	0.266822	0.266994	0.267025	0.26703	0.267031	0.267031	0.267031	0.267031	0.267031	0.267031	0.267031	0.267031
8	0.222251	0.240839	0.243598	0.244068	0.244151	0.244165	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168	0.244168
9	0.222706	0.231616	0.232938	0.233163	0.233202	0.233209	0.23321	0.233211	0.233211	0.233211	0.233211	0.233211	0.233211	0.233211	0.233211
10	0.222924	0.227195	0.227828	0.227936	0.227955	0.227958	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959	0.227959
11	0.223028	0.225075	0.225379	0.22543	0.22544	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441	0.225441
12	0.223078	0.22406	0.224205	0.22423	0.224234	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235	0.224235
13	0.223102	0.223573	0.223642	0.223654	0.223656	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657	0.223657
14	0.223114	0.223339	0.223373	0.223378	0.223379	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338	0.22338
15	0.223119	0.223227	0.223243	0.223246	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247	0.223247
16	0.223122	0.223174	0.223181	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183	0.223183
17	0.223123	0.223148	0.223152	0.223152	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153	0.223153
18	0.223124	0.223136	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138	0.223138
19	0.223124	0.22313	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131	0.223131
20	0.223124	0.223127	0.223127	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128	0.223128

SY-designs purple-circled have $\pi_L \leq 5\,ppm$; minimum-noise design has $\pi_L = 3.48ppm$

Thickness-Optimized SY-Coatings



$$L_{HWL}(H_{QWL}\ L_{QWL})^{N_{top}}(H'_{QWL}\ L_{QWL})^{N_{bot}}$$

➡ Optimum design for $\tau_P \le 5\,ppm$, $P_L \le 5\,ppm$ is $N_T = 1$, $N_b = 8$, featuring $\tau_P = 4.36\,ppm$, $P_L = 3.84\,ppm$, $R_\phi = 0.419$.

➡ **Explore thickness optimized version:**

Top stack: $\boldsymbol{\delta_H = 0.25 + \xi, \delta_L = 0.25 - \xi, \xi \in (0, 0.25)}$

Bottom Stack: $\boldsymbol{\delta_{H'} = 0.25 + \eta, \delta_L = 0.25 - \eta,\ \eta \in (0, 0.25)}$

(increasing ξ and/or η will decrease thermal noise)

Pick up (ξ, η) yielding minimum noise and acceptable loss, for some candidate $\{N_{top}\,,\ N_{bot}\,,\}$, keeping τ_P at the design value

Thickness Optimized SY-Coatings, contd.

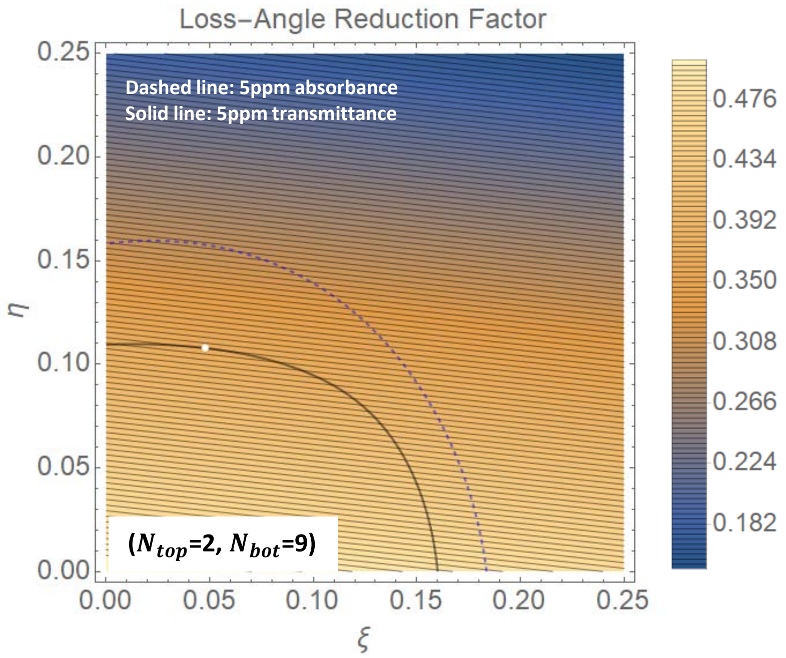
Noise PSD (coating loss-angle) reduction factor R_{ϕ}, thickness-optimized SY-Coatings:

N_{bot} N_{top}	5	6	7	8	9	10	11	12	13	14
1	-	-	-	.379	.381	...	...	...	...	...
2	-	-	.383	.367	.364	.365	...	...	...	...
3	-	.446	.400	.390	.387	.386	.387	.389	...	...
4	-	.446	.428	.421	.419	.417	.417	.418	.419	...
5	-	.472	.460	.455	.452	.451	.451	.451	.452	...
6	.525	.503	.494	.490	.487	.486	.486	.486	.487	...

- The QWL (1,8) SY-design features a coating loss-angle reduction factor (compared to the reference 17-doublets design) R_{ϕ}=0.419.
- A few thickness-optimized designs (highlighted in cyan in the Table) can do better on the first decimal figure (improvements at the level of the 2nd decimal figure and beyond would be likely spoiled by deposition errors).
- The best thickness-optimized SY-design has

  (N_{top}=2, N_{bot}=9), (ξ=0.0475, η= 0.1079)

  with a noise reduction factor R_{ϕ}=0.364.



Conclusions

- Stacked-triplet coatings based, e.g., on Silica, (Ti::)Tantala and *a*Silicon are easily designed;
- Generalization to more than 3 materials is straightforward in principle, in view of Larruquert rule;
- At 1550nm and 10K a stacked-triplet aSilicon/Ti::Tantala/Silica coating with $\tau_P = 5\,ppm$ has a coating loss angle (noise PSD) smaller by a factor $R_\phi = 0.406$ compared to the reference $5\,ppm$ Silica/Ti::Tantala design, slightly better compared to the best SY-coating ($R_\phi = 0.419$), with an absorbance of 4.65ppm (vs 3.85ppm for the SY-design);
- Thickness-optimization improves the performance of SY-ternary coatings; for the ET-LF like case considered here the coating noise PSD reduction factor R_ϕ drops from 0.419 to 0.368;
- Further analysis/comparison, in terms of, e.g., spectral response flatness, robustness vs. deposition tolerances, coating stress distribution etc. is in order;
- Whole new world of options (triplet/doublet and multiplet compounds) !

Acknowledgement

This work has been developed in part within the tuition activity of MIT student ***Rokas Veitas****, during his stay at USannio in January 2020, in the frame of the MIT - USannio externship program. Rokas contribution, and many stimulating discussions with him are kindly acknowledged .*

Benevento, IT Jan 20th 2020

Questions/Comments Welcome !