Red Noise in the 12p5-Year NANOGrav Data Set

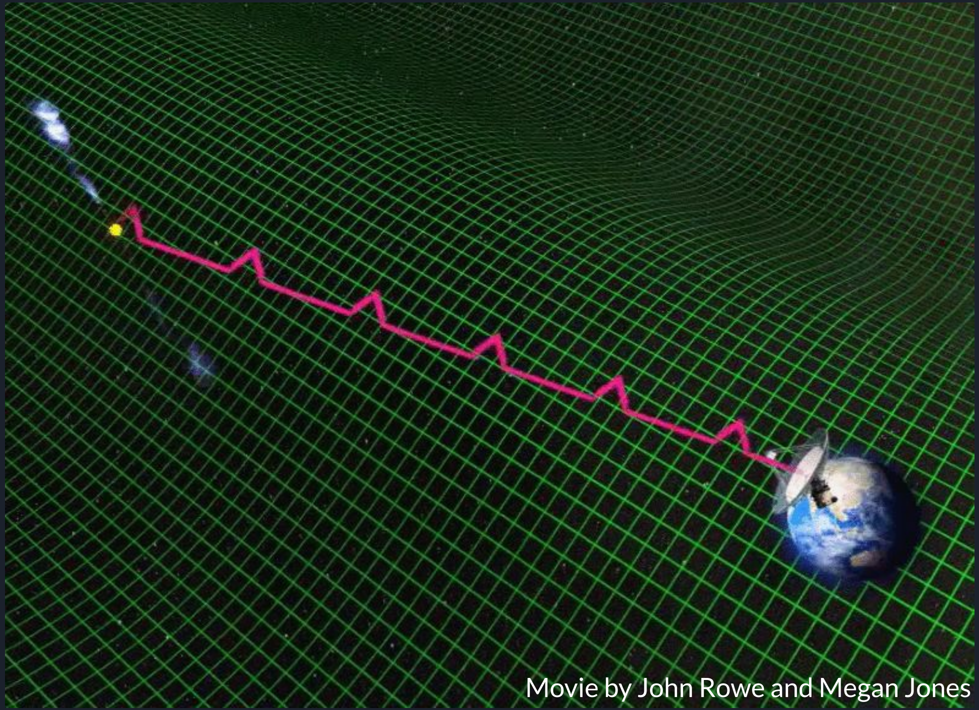
Jerry Sun

How can we see gravitational waves?

Millisecond pulsars (MSPs) have stable rotation rates

GWs cause objects to oscillate towards and away from each other

The oscillation causes pulses to arrive too early or too late



How can we see gravitational waves?

Array of millisecond pulsars (MSPs)

Detect GWs with frequencies of years-decades

Stochastic Gravitational Wave Background (SGWB)

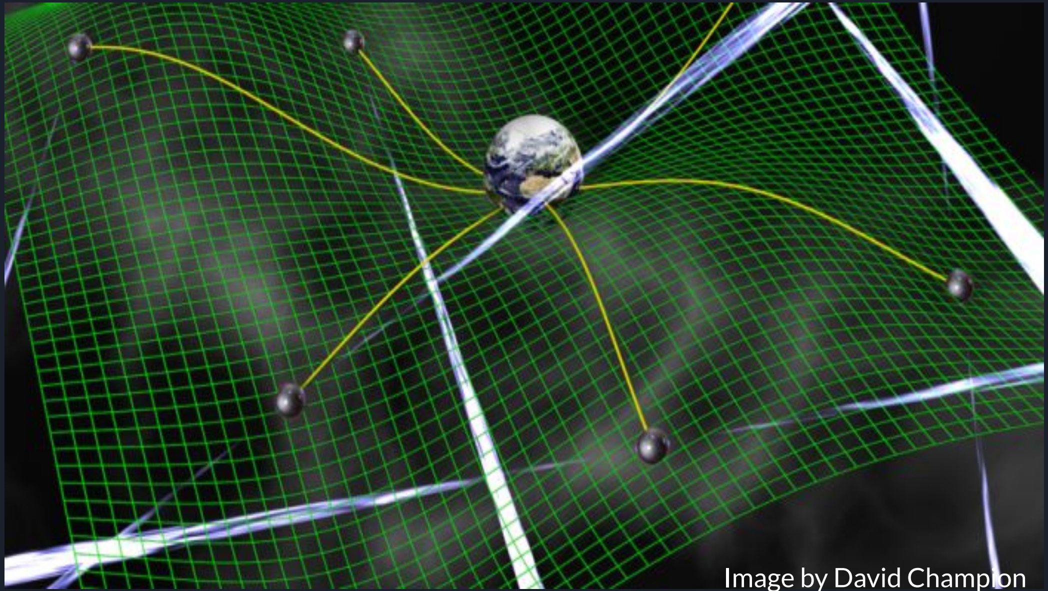


Image by David Champion

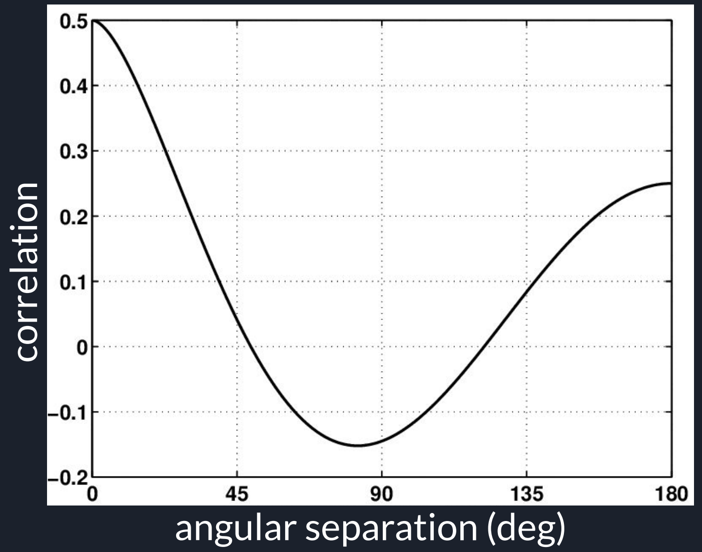
What constitutes a detection?

Detection of a SGWB is evidenced by:

1. A significant common process with a red power spectral density with γ=13/3 (often called common red noise or cRN)

$$p(f) = Af^{-\gamma}$$

2. Hellings-Downs angular correlation pulse time-of-arrivals (TOAs) from pairs of pulsars



R. W. Hellings and G. S. Downs, Astrophys. J. Lett. 265, L39 (1983).

Proof in the Pudding - Bayes' Theorem

Bayes' Theorem allows us to estimate model parameters given data

$$pr(\theta|\text{data}) = pr(\text{data}|\theta) * pr(\theta)$$

posterior — likelihood — prior

Under a uniform prior, the posterior probability is just proportional to the likelihood!

Sampling the Posterior Distribution

We use a Markov Chain Monte Carlo sampler to sample the posterior.

$$A(\theta \to \theta') = \frac{pr(\theta'|d)}{pr(\theta|d)} \frac{pr(\theta' \to \theta)}{pr(\theta \to \theta')}$$

A Methodology for a Real Dataset

$$\Sigma = \begin{bmatrix} P_1 & S_{12} & \dots & S_{1M} \\ S_{21} & P_2 & \dots & S_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ S_{M1} & S_{M2} & \dots & P_M \end{bmatrix} \quad \begin{array}{l} P_I = \langle r_I r_I^T \rangle \\ S_{IJ} = \langle r_I r_J^T \rangle |_{I \neq J} \end{array}$$

$$p(r|\theta) = \frac{1}{\sqrt{\det(2\pi\Sigma)}} \exp\left(-\frac{1}{2} r^T \Sigma^{-1} r\right)$$

Siemens, Xavier, et al. "The Stochastic Background: Scaling Laws and Time to Detection for Pulsar Timing Arrays." Classical and Quantum Gravity, vol. 30, no. 22, 2013, p. 14.c

Methods: Modeling the GWB

Recall that common red process may indicate the presence of a gravitational wave

$$p(f) = Af^{-\gamma}$$

Red-noise process	1	2A
intrinsic (per pulsar)	x	x
uncorr. common		x
H.-D Common		
dipole corr.common		
monopole corr.common		

Arzoumanian, Z., et al. "The NANOGrav 11 Year Data Set: Pulsar-Timing Constraints on the Stochastic Gravitational-Wave Background." The Astrophysical Journal, vol. 859, no. 1, 2018, p. 22.

Results - First Look (Model 1)

Model 1: *per pulsar intrinsic noise* only

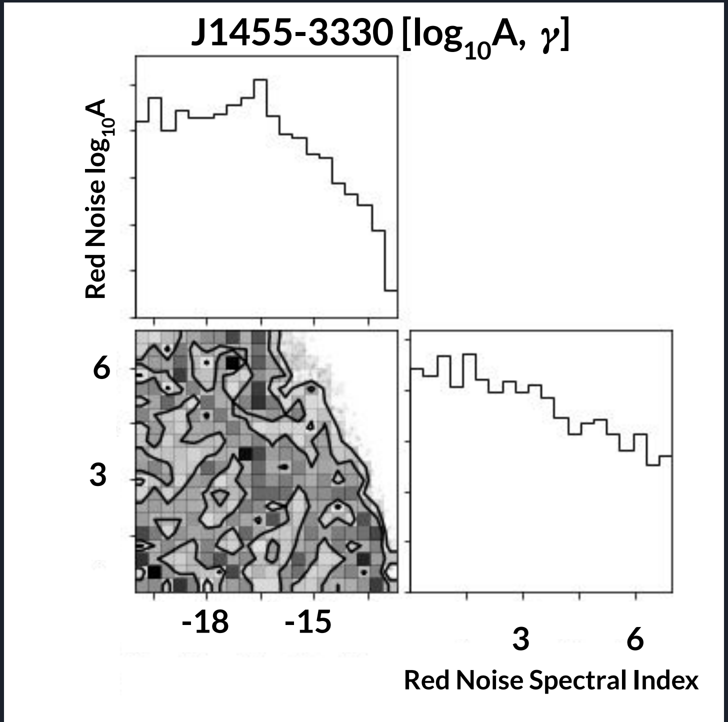
$$\Sigma_1 = \begin{bmatrix} P_1 & 0 & \dots & 0 \\ 0 & P_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_M \end{bmatrix}$$

Results - First Look (Model 1)

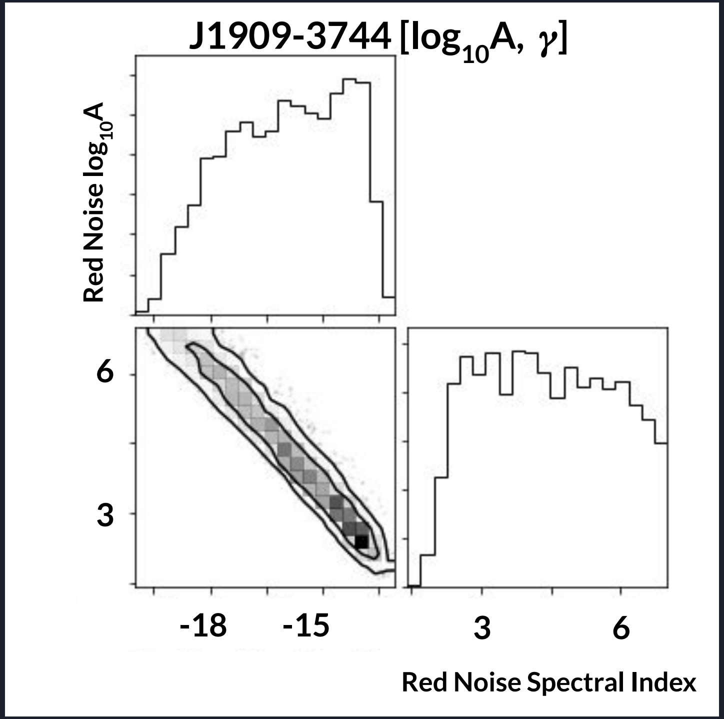
We found that it was easiest to separate our pulsars into three major categories:

1. Pulsars that showed *no evidence* of a red process
2. Pulsars that showed *strong evidence* of a red process
3. Weirdo pulsars that we don't understand

Ex. Uninformative Pulsars



Ex. Pulsars Displaying a Red Process

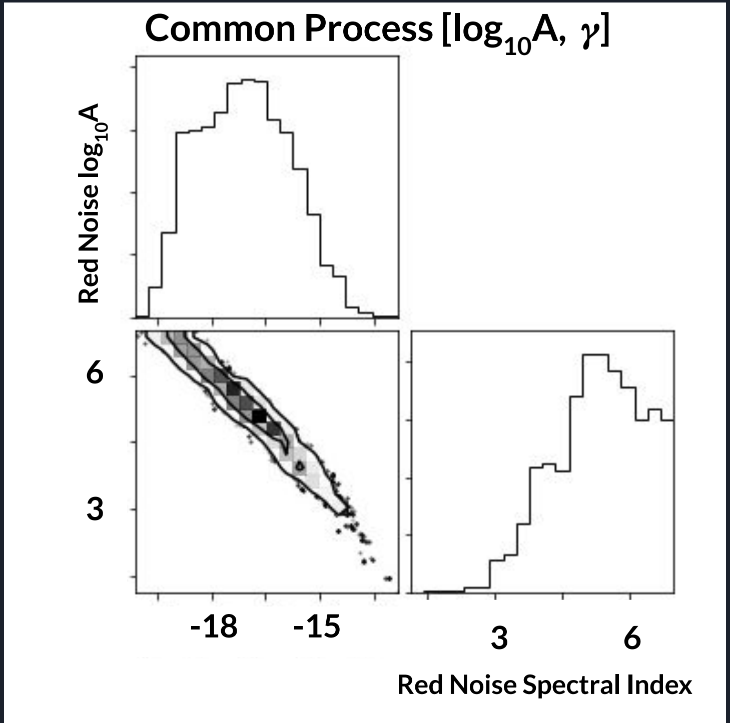


Results - Diving Deeper (Model 2A)

Model 2A: *per pulsar intrinsic noise* along with a common red process

$$\Sigma_{2A} = \begin{bmatrix} P_1 + cRN & 0 & \dots & 0 \\ 0 & P_2 + cRN & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_M + cRN \end{bmatrix}$$

Results - Diving Deeper (Model 2A)

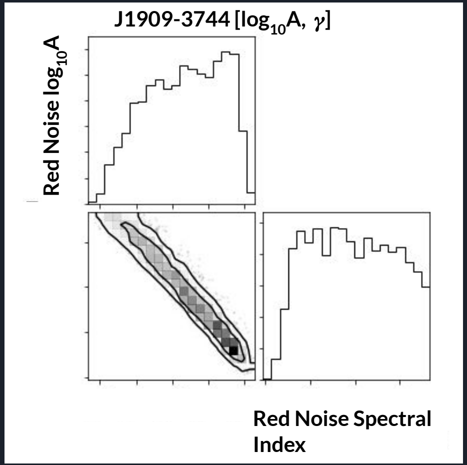


Conclusion: it looks like there's evidence for a strong red-noise signal in the data!

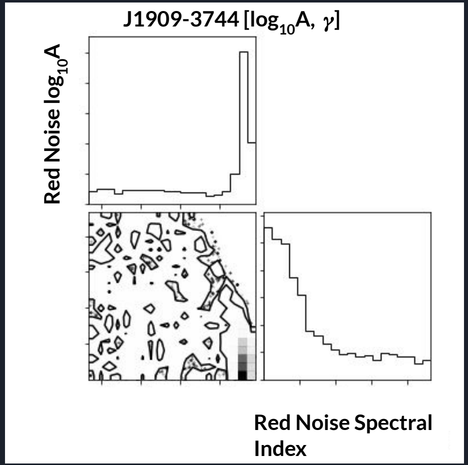
Results - Diving Deeper (Model 2A)

Some of the intrinsic red noise is “absorbed” into the common red process, and disappears from the individual:

Model 1



Model 2A



Results - Diving Deeper (Model 2A)

Other NANOGrav analyses agreed that there were some (but not a lot of) pulsars that showed evidence of a common red noise.

Pulsars Contributing To the Common Process

Overview

Dropout Method

Favor cRN:
- J2043
- J1911
- J1741
- J0613
- J2234
- J1909

Favor cRN, 13/3:
- J2043
- J1744

Disfavor cRN:
- J1713

Comparing Per Pulsar RN

“Bleeds” into cRN:
- J0030
- J0613
- J1713
- J1909
- J2043
- J2317

Strange Behavior:
- B1855
- J1744
- J2145

Likelihood Factorization

Large BF for 13/3 cRN:
- J1909
- B1855
- J0030
- J2317
- J2043
- J1744
- J0613

Summary

Gravitational Waves can be detected by precise pulsar timing

We use a Monte Carlo sampler to estimate the GWB parameters

Using simpler models, it's not yet conclusive whether or not there is a GW in our data. However, there is a 10,000:1 ratio that supports the existence of at least uncorrelated red noise.

The full analysis has been completed, and the Bayes' Factor for an HD-correlated red process to an uncorrelated process is about 3