NCal Penultimate Mass Contribution

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1 Introduction

Since the Newtonian calibrator (NCal) uses gravity to inject calibration lines, one concern is that there is a force applied to the entire quadruple pendulum. Most notable, the NCal pulls on the Penultimate Mass (PUM) which can cause displacements of the Test Mass (TM).

In this note, we describe calculations which yield the expected shifts due to this path, NCal-to-PUM-to-TM.

2 FEA Calculation

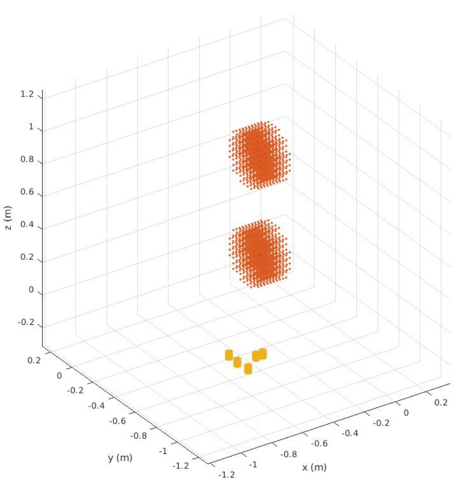

Figure 1: Point mass representation of the NCal, the PUM, and the TM.

We extended the Finite Element Analysis developed for the NCal force predictions [1] to include the PUM. This analysis breaks the geometry up into a cloud of point masses, shown in Figure 1, and then sums up the force between each point pair.

This treats the PUM as a carbon copy of the TM just shifted up by 601.98 mm [2, 3]. The code then outputs a separate force for both the TM and the PUM. All of the input parameters of the system are then varied in a Monte Carlo calculation to yield a force distribution for each.

3 Transfer Function

To propagate the force acting on the PUM to the TM, we use the known Force-to-Length transfer functions [2, 3]. These are shown in Figure 2.

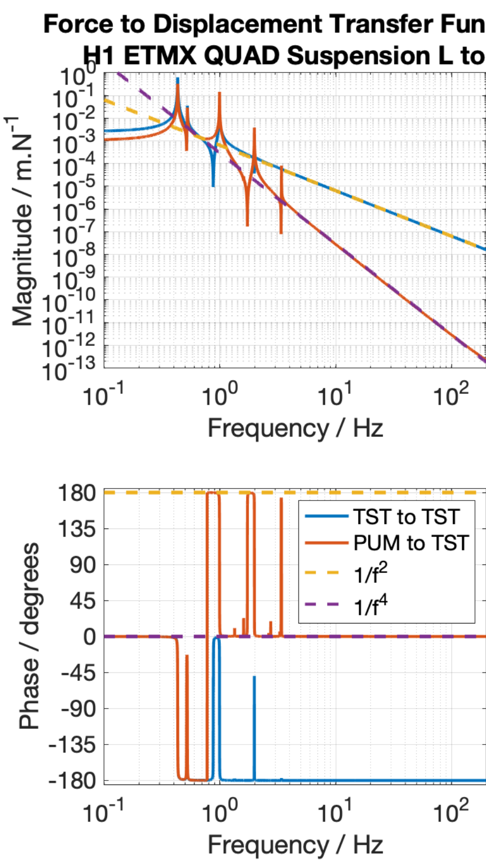

Figure 2: Force to length transfer functions.

We need force on the PUM to force on TM but only have force to displacement transfer functions. So we take the ratio of the PUM force to displacement and the TM force to displacement to yield a force to force transfer function.

$$T_{\text{F-to-F}}^{\text{PUM-to-TM}} = \frac{T_{\text{F-to-x}}^{\text{PUM}}}{T_{\text{F-to-x}}^{\text{TM}}} \quad (1)$$

4 Results

The PUM contribution is then added to the direct TM force at each frequency. The TM force is frequency independent but, due to the frequency dependent transfer functions, the PUM contribution is not. The results of these calculation are shown in Figures 3 and 4 for the quadrupole and hexapole mass configurations.

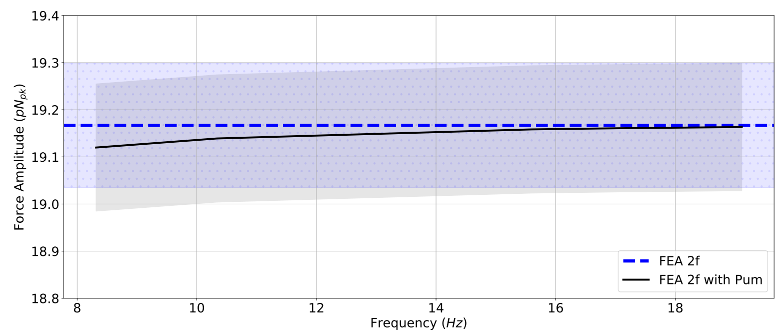

Figure 3: Force predication for the quadrupole force with and without the PUM

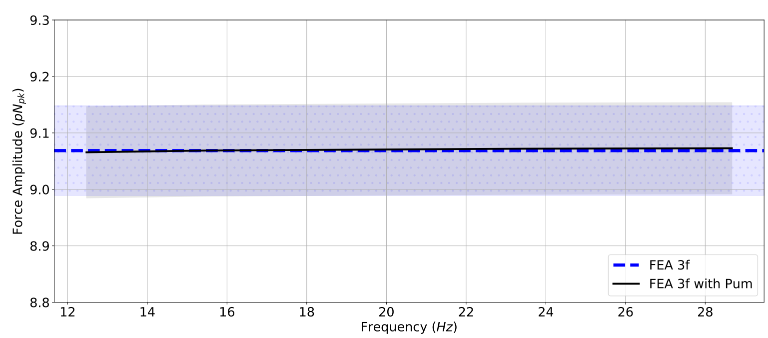

Figure 4: Force predication for the hexapole force with and without the PUM

The relative shift due to the PUM contribution is shown in the following Table and ranges from 0.245 % at the lowest frequencies for the 2f and 0.002% at the higher frequencies.

$2f$ (Hz)	F_x^{2f} PUM	F_x^{2f} Meas	$3f$ (Hz)	F_x^{3f} PUM	F_x^{3f} Meas
8.32 Hz	0.279 %	not used	12.45 Hz	0.046 %	3.39%
10.34 Hz	0.178 %	14.02%	15.51 Hz	0.030 %	4.81%
15.60 Hz	0.078 %	1.51%	23.40 Hz	0.013 %	3.90%
17.11 Hz	0.065 %	2.41%	25.66 Hz	0.011 %	6.07%
19.11 Hz	0.052 %	1.09%	28.67 Hz	0.009 %	2.52%

Table 1: Shifts due to the PUM relative to the predicted force, F_x^i PUM, along with the current measurement uncertainties at each frequency, F_x^i Meas, for both the quadrupole and hexapole mass arrangements.

5 Conclusion

Note that the decreased influence of the NCal on the PUM at $3f$ is expected due to the $3f$ force falling off as $1/d^5$ where as the $2f$ falls like $1/d^4$.

When compared to the current systematic uncertainty [1] of 0.7%-0.9% these shifts are well within the uncertainty bounds for most frequencies. These shifts are even more negligible when compared to the current measured force uncertainties of $\sim 1\% - 14\%$.

References

[1] P1900244.

[2] Mark Barton. T020205.

[3] Mark Barton. T080188.