Understanding the Physical Degrees of Freedom in a Parameterized Test of General Relativity

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Introduction to Project

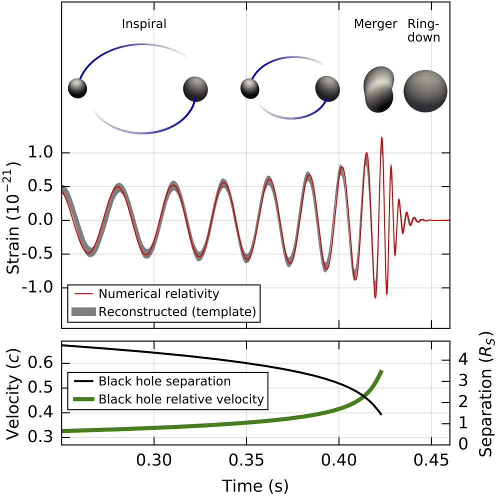

Figure 1 – Results of GW150914 (Abbott *et al.*, 2016).

Goal of the Project

- The goal of this research project is to provides a framework for understanding the **physical degrees of freedom** in a **parameterized test** of general relativity (GR).
- Particularly, we would like to vary the **post-Newtonian (PN) coefficients**, the **phenomenological coefficients**, and the **analytical black-hole perturbation theory waveform parameters**, and observe how this would affect the waveform and hence the physical parameters.

Parameterized Test of GR

- In particular, we use `IMRPhenomPv2` to carry out the parameterized test of GR. `IMRPhenomPv2` (Khan *et al.*, 2016, 2019; Hannam *et al.*, 2014; Husa *et al.*, 2016).
- To perform the parameterized tests, we introduce fractional deviations δp_i to the `IMRPhenomPv2` phase coefficients p_i (Li, 2013), namely

$$p_i \to (1 + \delta p_i) p_i. \tag{3.1}$$

Stage of Coalescence	Dephasing Coefficient (δp_i)
Inspiral	$\{\delta\chi_0, \ldots, \delta\chi_7\}$
Intermediate	$\{\delta\beta_2, \delta\beta_3\}$
Merger-Ringdown	$\{\delta\alpha_2, \ldots, \delta\alpha_5\}$

Parameterized Test of GR

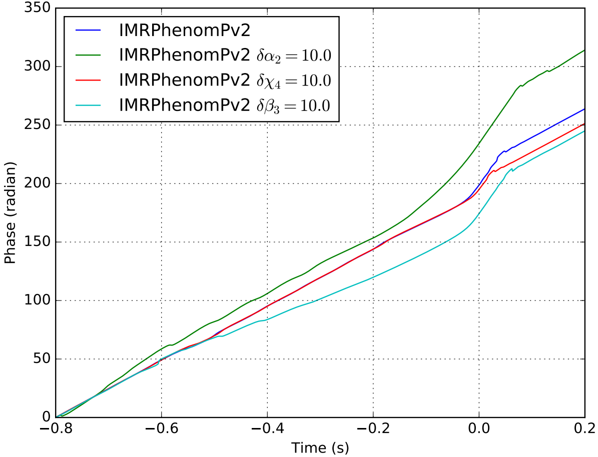

Figure 2 – Phase of GW versus time for `IMRPhenomPv2` with no modification, $\delta\alpha_2 = 10.0$, $\delta\chi_4 = 10.0$, and $\delta\beta_3 = 10.0$.

Parameterized Test of GR

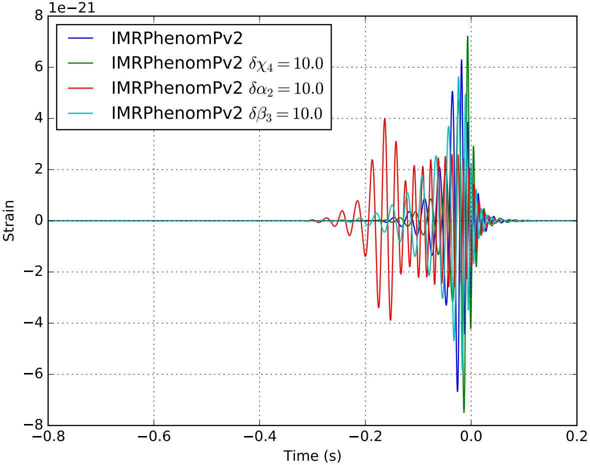

Figure 3 – Strain versus time for `IMRPhenomPv2` with no modification, $\delta\alpha_2 = 10.0$, $\delta\chi_4 = 10.0$, and $\delta\beta_3 = 10.0$.

A Multipole Expansion of Energy

- To compute the energy carried by a GW, we take the 00-component of the Isaacson stress-energy tensor and integrate over the volume V (Maggiore, 2007; Ruiz *et al.*, 2008)

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \lim_{r\to\infty} \frac{1}{16\pi} \int_S \mathrm{d}\Omega\, r^2 \left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle , \tag{4.1}$$

- To obtain an analytic expression for the integral over solid angle Ω, we separate h_+ and $h_\times$ into a time-dependent part and an angular part.
- This can be done using spin-weighted spherical harmonics ${}_sY_{\ell m}$.

$$\begin{aligned} h_+ - ih_\times &= \sum_{l,m} {}_{-2}Y_{lm}(\theta,\phi) h_{l,m}(t) \\ &\approx {}_{-2}Y_{22}(\theta,\phi)h_{2,2}(t) + {}_{-2}Y_{2-2}(\theta,\phi)h_{2,-2}(t). \end{aligned} \tag{4.2}$$

A Multipole Expansion of Energy

- Now, we can integrate over solid angle to obtain the instantaneous power

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \lim_{r\to\infty} \frac{r^2}{16\pi} \left\langle |\dot{h}_{2,2}|^2 + |\dot{h}_{2,-2}|^2 + \frac{1}{6}\left(\dot{h}^*_{2,2}\dot{h}_{2,-2} + \dot{h}_{2,2}\dot{h}^*_{2,-2}\right)\right\rangle . \quad (4.3)$$

- To avoid calculating the average over several wavelengths, one approach is to numerically calculate the cumulative energy which is simply

$$E = \int_{-\infty}^{t} dt' \frac{\mathrm{d}E}{\mathrm{d}t'} \quad (4.4)$$

Effects of Varying the Intrinsic Parameters

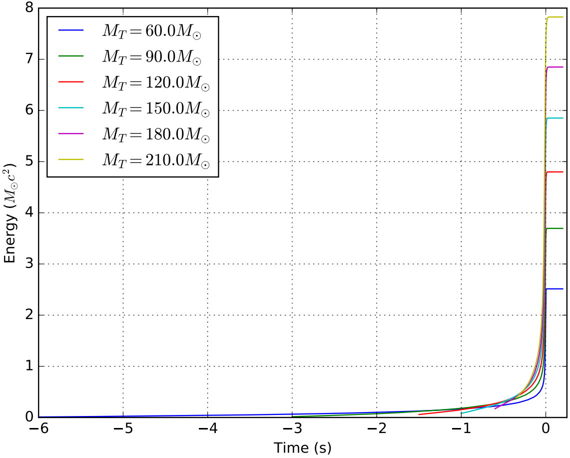

Figure 4 – The cumulative radiated energy of GW versus time in linear scale for `IMRPhenomPv2` with constant mass ratio $q = 1.00$ and varying total mass.

Effects of Varying the Intrinsic Parameters

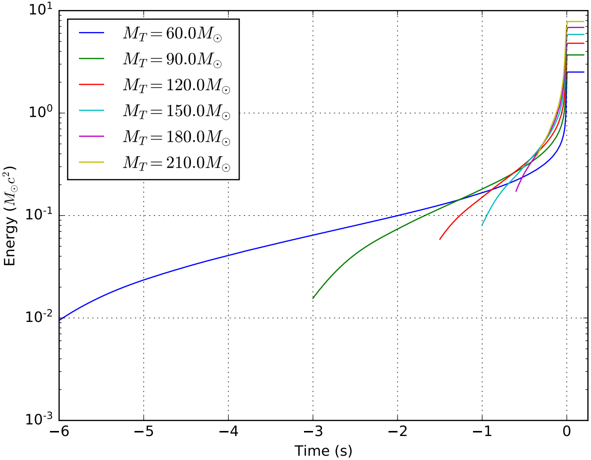

Figure 5 – The cumulative radiated energy of GW versus time for IMRPhenomPv2 with constant mass ratio $q = 1.00$ and varying total mass in logarithmic scale.

Effects of Varying the Dephasing Coefficients

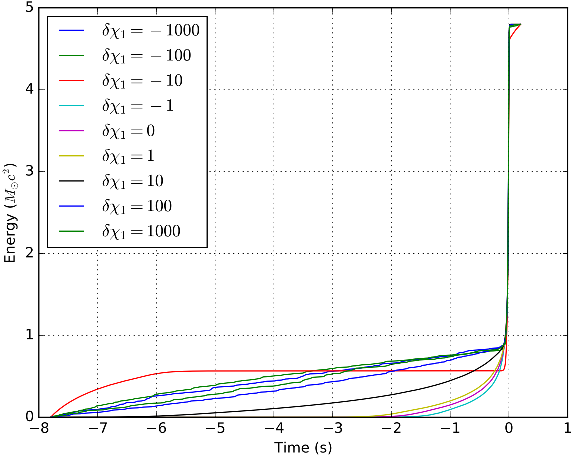

Figure 6 – Cumulative energy of GW versus time with $m_1 = m_2 = 60\ M_\odot$, no spin, and varying $\delta\chi_1$.

Effects of Varying the Dephasing Coefficients

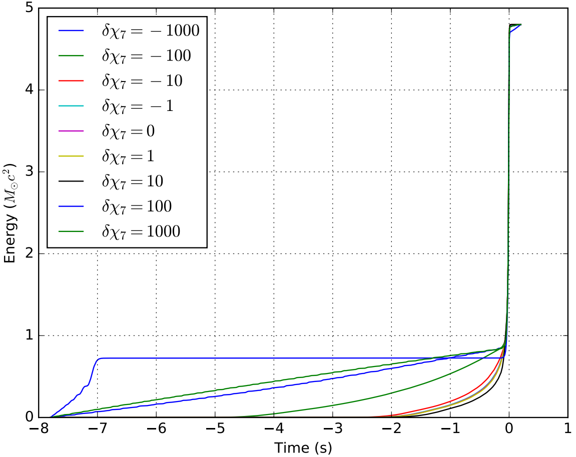

Figure 7 – Cumulative energy of GW versus time with $m_1 = m_2 = 60\ M_\odot$, no spin, and varying $\delta\chi_7$.

Effects of Varying the Dephasing Coefficients

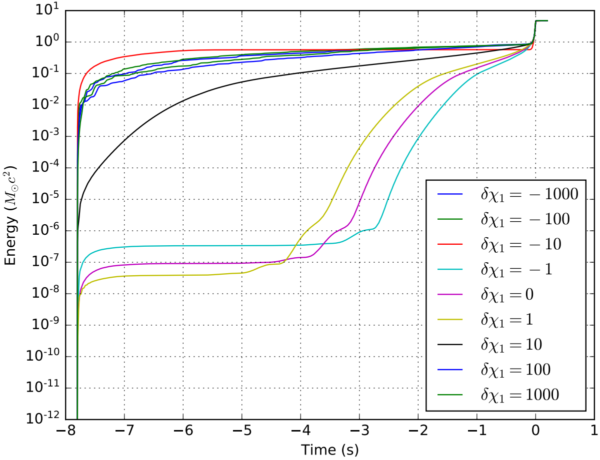

Figure 8 – Cumulative energy of GW versus time with $m_1 = m_2 = 60\ M_\odot$, no spin, and varying $\delta\chi_1$ in logarithmic scale.

Effects of Varying the Dephasing Coefficients

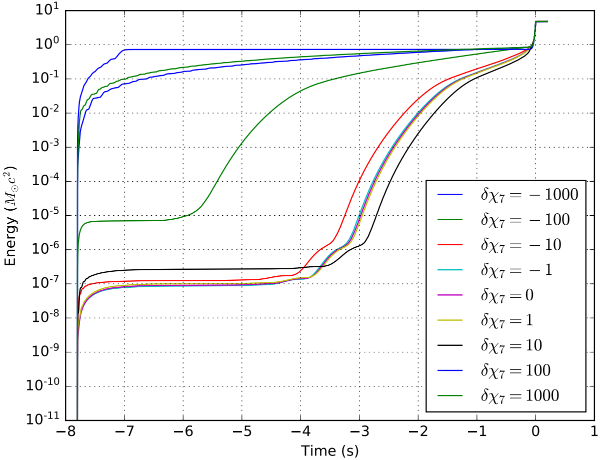

Figure 9 – Cumulative energy of GW versus time with $m_1 = m_2 = 60\ M_\odot$, no spin, and varying $\delta\chi_7$ in logarithmic scale.

Effects of Varying the Dephasing Coefficients

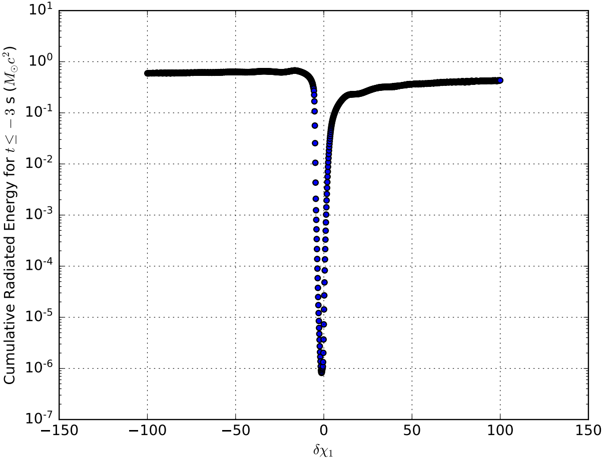

Figure 10 – Cumulative radiated energy of GW for times $t < -3$ s versus $\delta\chi_1$ with $m_1 = m_2 = 60\ M_\odot$ and no spin in logarithmic scale.

Effects of Varying the Dephasing Coefficients

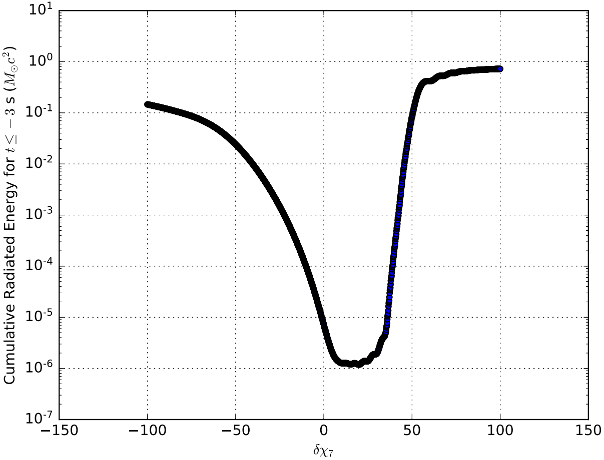

Figure 11 – Cumulative radiated energy of GW for times $t < -3$ s versus $\delta\chi_7$ with $m_1 = m_2 = 60\ M_\odot$ and no spin in logarithmic scale.

Future Work

- We will focus on extending similar analyses provided in this project to the intermediate and merger-ringdown phases.
- We will then vary the intrinsic parameters (such as total mass, mass ratio, and spin) and the dephasing coefficients to see how they affect the evolution of angular momentum.
- We plan to use these physical quantities, namely energy, and angular momentum, to constrain the dephasing parameters.

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