LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
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Characterization and Control of the Auxiliary Laser PZT

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Contents

1 Abstract

The auxiliary lasers in 40m are used to stabilize arm lengths during full lock acquisition. The laser frequency is stabilized to the length of the arm cavities using the Pound–Drever–Hall (PDH) scheme. In this particular setup, the phase modulation is done by exploiting a mechanical resonance in the laser's piezoelectric transducer (PZT) which also works as the controller actuator. The PZT mechanical resonances restrict the maximum bandwidth to which we can control the laser frequency due to oscillations and excessive phase delay. The aim of this project is to accurately measure the open-loop transfer function of the auxiliary laser PDH loop and identify the poles and zeroes of the system by fitting the data. With this done, the inverse of the transfer function can be found and applied to the system using digital filters, effectively canceling out the mechanical resonances of the PZT and thus increasing the bandwidth of the system. It will add a digital aspect to the laser control systems that were previously entirely analog.

2 Introduction

The Laser Interferometer Gravitational-Wave Observatory (LIGO), detects and observes gravitational waves, and thereby colliding black holes and neuron stars, and supernovae. The LIGO interferometer is a Michelson interferometer, a precision instrument that splits a light beam into two parts and recombines them after they have traveled different optical paths, to produce interference fringes. The Fabry-Pérot cavity is an optical cavity consisting of two reflecting mirrors wherein optical waves can only pass through the cavity when they are in resonance with it. The LIGO interferometer is modified with Fabry-Pérot cavities. Advanced LIGO (aLIGO) uses these cavities to increase the power of the laser beam and enhance the signal received by the detector. Power is stored in the cavities as they are locked to the main laser.

The entire detector system consists of 5 degrees of freedom all of which must be controlled simultaneously in order to bring the detector to the observation state. The purpose of the Arm Length Stabilisation is to stabilize the cavity length motion with respect to the main laser frequency while keeping it off-resonance. Frequency-doubled lasers injected on either end of the interferometer are used to acquire arm stabilization during locking. The system is locked using the Pound-Drever-Hall (PDH) technique, which reads a feedback signal to a controller which takes the error signal and converts it to a voltage that is fed back to the laser, keeping it locked and resonant with the cavity.

The initial goal of the project is to characterize the laser PZT's control loop. The unity gain frequency UGF is around 11kHz; ideally, we would increase it to anywhere from 10 to 100 times that value. This effectively increases the bandwidth of the system and its ability to suppress noise at lower frequencies, ensuring its operation with maximized sensitivity. In the future, an instrument can be set up to automatically perform system identification, and an FPGA can be programmed to perform the inversion of the measured resonances.

3 Feedback and Controls: Auxilliary Laser Loop

The use of algorithms to sense, compute and actuate dynamic systems is integral to control. In control systems, often a certain variable is used to evaluate the performance of the system. This variable is measured and compared to the desired value, and accordingly, the control system attempts to correct this error using the actuators. This is known as feedback and may be broadly defined as the interaction between two (or more) dynamic systems. A feedback system forms when the output of the system feeds back into the input. The cancellation of the error signal is best at low frequencies where the actuator response is fast enough to not let the error go high. Beyond the actuation limit, at high frequencies, the cancellation does not work anymore. This defines the bandwidth of the feedback loop.

A transfer function models a system as the relationship between its inputs and outputs. The transfer function may specifically be described in the frequency domain, in which case a Fourier or Laplace transform is performed. We use the transfer function of a system to characterize the system and understand its response to various inputs.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\mathfrak{L}(y(t))}{\mathfrak{L}(x(t))}$$

The gravitational wave detector requires precise control to stabilize the lasers. The Pound-Drever-Hall technique is a method used to stabilize the light emitted by a laser by locking the light to a cavity. The servo is employed to move the mirrors until the interferometer is at a desirable configuration. The preliminary objective of the project is to increase system bandwidth and subsequently suppress the noise of the system at lower frequencies. The completion of this objective enables us to work towards finding a new calibration method for the LIGO detectors with less systematic uncertainty. The loop consists of the PDH sensor (a photodiode reads the PDH error signal which is demodulated by a mixer and low passed), the uPDH servo box which performs the feedback filters and amplification, and the Laser PZT which is the actuator.

4 System Identification

System identification involves the use of statistical methods to model systems mathematically. In simpler terms, it is using experimental data obtained from the input/output relations to model dynamic systems. In our case, the system at hand is the PZT actuator of the auxiliary laser.

1. **Red Pitaya and Spectrum Analysis**

   In order to get comfortable with system identification, the study of the Red Pitaya, a System-on-Chip was carried out, in an attempt to characterize the various noise involved in signal acquisition. A particular focus was the performance evaluation of the platform's ADCs and DACs. The Red Pitaya has its own signal generators but in order to get an unbiased characterization, external signal generators were used.

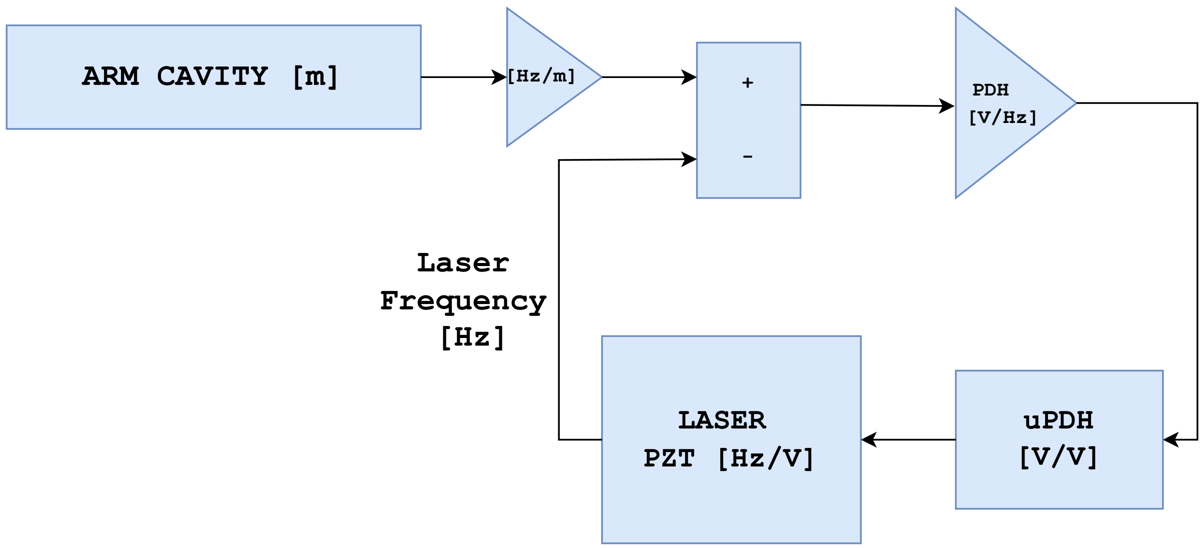

Figure 1: Thematic diagram of the AUX Loop

The noise in a Red Pitaya is environmental as well as intrinsic, for example, thermal noise, external noise from cellular interference, shot noise, and Johnson–Nyquist noise. Noise can also occur due to fluctuations in the reference voltage source or the clock jitter of the FPGA. This was an important exercise in familiarising myself with the basics of electronic system characterization and the software interface required for the same. Usually, ADC noise may be measured with either an input short test or a sine wave input test. Both tests are performed, however, the sine wave test is more important to us as it is usually used for high-speed data acquisition.

2. **Frequency Response and Transfer Functions**

   The frequency response of different filters is collected and used to find out the transfer function of the filter using a software known as vectfit. Vectfit provides us with the necessary information to accordingly deduce the various components of the circuit. We can also find the state space variables, or poles and zeros of the system if necessary.

   The same concept is applied as we begin to characterize the control loop of the laser, we excite the circuit (with a swept sine wave) and begin to understand the transfer function, the various noises being detected, and the gain parameters of the system. With an understanding of the exact limitations of the system, we can proceed to try and improve the performance of the circuit, with a particular focus on increasing the gain itself, as this gives us the relationship between input and output magnitude of the system at steady state. A higher gain for the auxiliary laser control loop is desirable.

5 Experimental Setup

I set up a MokuLab, a multi-instrument that behaves as a phasemeter and signal generator in an attempt to read the transfer function of the loop formed by the PZT which controls the laser and the resultant movement in the laser system. The input is a swept sine wave and

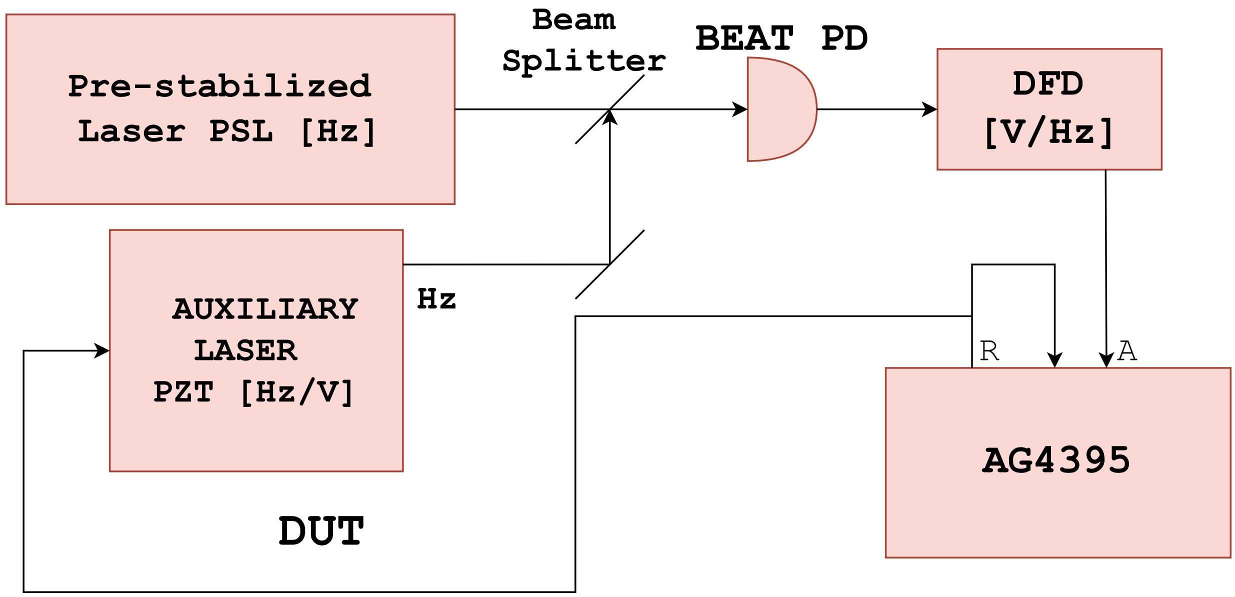

Figure 2: Experimental Setup using the Delay Line Frequency Discriminator [4]

the output is the frequency of the beatnote formed by the auxiliary and main lasers. This system was extremely convenient and was capable of measuring the fluctuations in beatnote frequency. However, it was not capable of inserting the required frequency of swept sine signal into the circuit, which would enable us to observe the beatnote frequency fluctuations at higher frequencies. [The maximum available frequency of the Moku:Lab was 10kHz, and we required a frequency of nearly 30kHz.]

A phase-locked loop is a control system in which the output signal generated has a phase related to that of the input signal. The simplest phase-locked loop may be thought of as a voltage-controlled oscillator, a low-pass filter, and a phase detector. The oscillator generates a frequency and the phase detector detects the phase of the output, the error generated is used by the VCO to correct the output phase signal, and attempts to match it to that of the input. I set up a phase-locked loop including a voltage-controlled oscillator, the Marconi IFR 2023b. The system worked smoothly and was quite stable, but did not have the bandwidth to track the auxiliary laser's beatnote with the PSL, which moves with frequencies in the megahertz.

A delay-line frequency discriminator (DFD) was set up to measure the frequency fluctuations of the auxiliary laser with respect to the PSL (beatnote). In the delay (line) frequency discriminator method, an optical beat is generated, and the beatnote signal is preamplified and split into two paths. One path is applied to the LO port of a passive double-balanced mixer acting as a phase detector. The other path is passed through a delay line and then through a phase shifter. The mixing of the RF signal with a delayed version of itself generates a voltage signal from which we can extract the frequency fluctuations of the original signal. Figure 3 shows the set-up employed to take measurements.

The DFD enables us to generate a signal directly proportional to the fluctuations of the beat frequency. For our purpose, however, the beatnote provides us with an output for

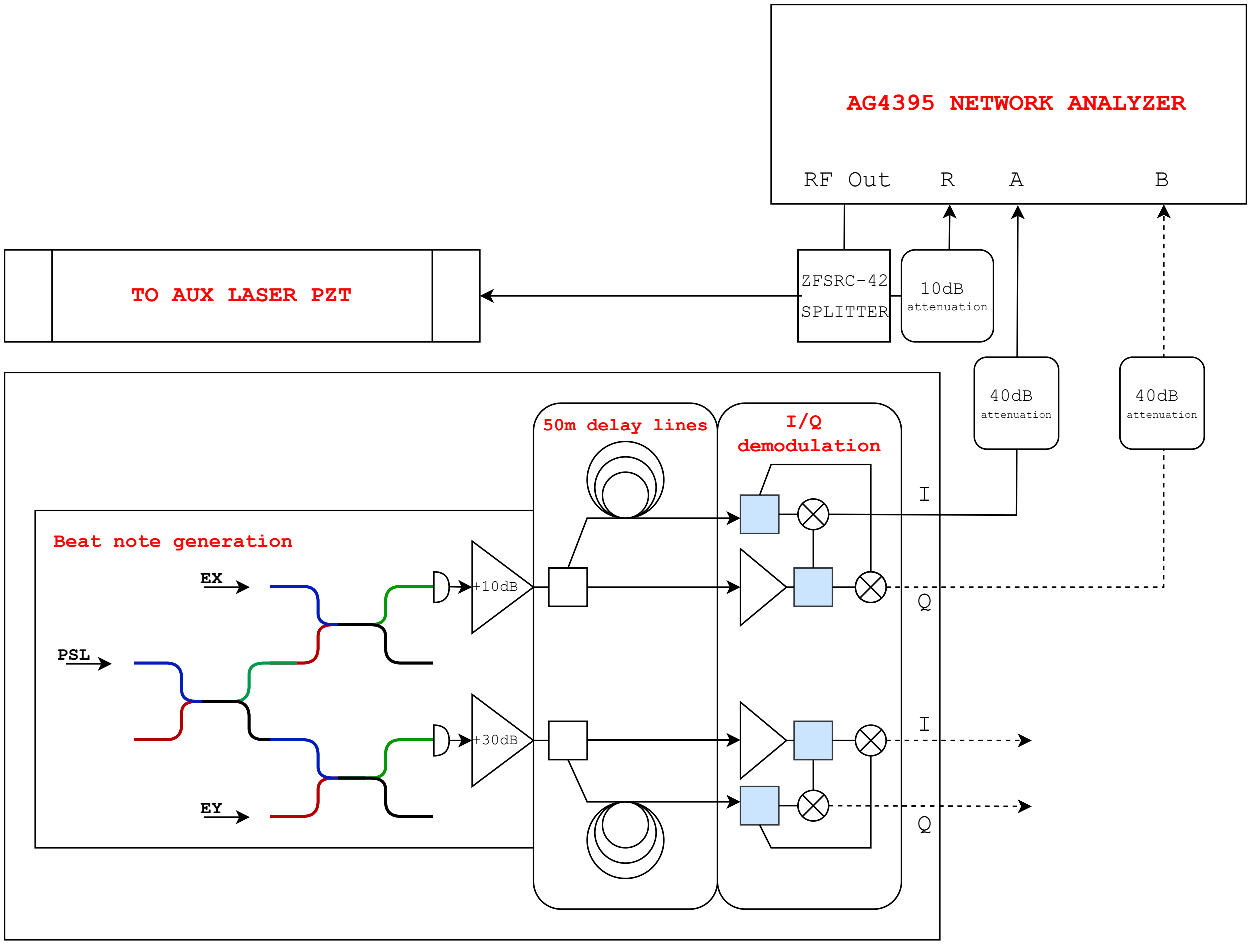

Figure 3: Detailed Schematic of Delay Line Frequency Discriminator [3]

the excitation we added with the purpose of measuring the transfer function. We plot the frequency response as the DFD output for respective frequency excitation, as recorded on a network analyzer. In this manner, the delay-line enables us to track the beatnote's high-frequency fluctuations where the other methods fail.

6 Measurements and Analysis

1. **Frequency Discriminator Transfer Response**

   An important part of analyzing the signal output by the DFD was calculating the DFD discriminant so we could accurately evaluate the data we had collected. Essentially, this involves understanding the transformation process from the frequency fluctuations to the analog voltage signal and applying this numerical analysis to arrive at an accurate measure of frequency fluctuations. The process of arriving at this discriminant involves considering the effects of the various electronic components such as splitters, mixers, and filters, the intentionally introduced delay, and assuming the quadrature rule and small signal model are valid. The following formula from [6] takes all of these factors

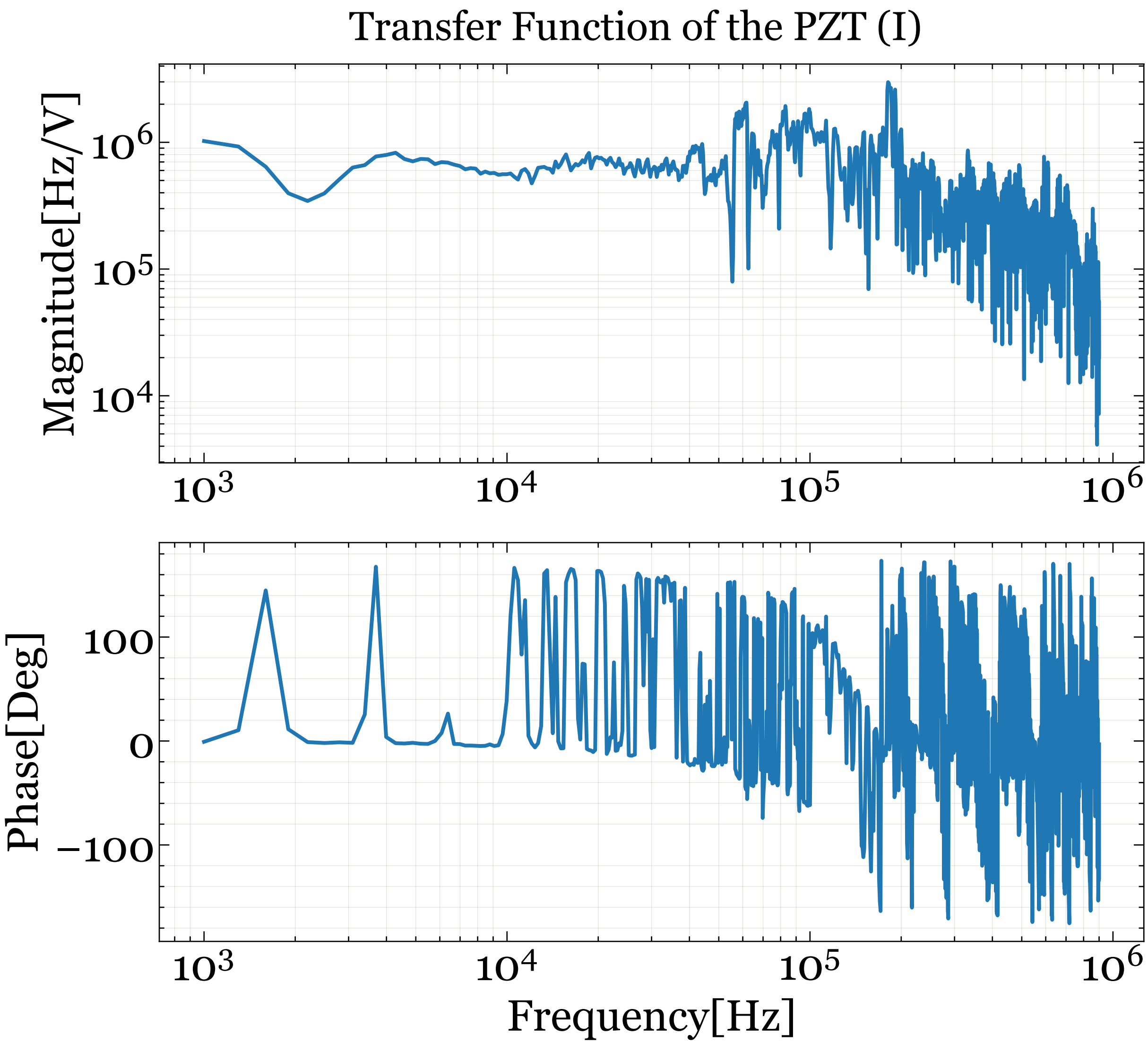

Figure 4: PZT Transfer Function as measured by the DFD

into consideration

$$\Delta V(f_m) = K_\phi 2\pi \Delta f(f_m) \frac{sin(\pi f_m \tau_d)}{\pi f_m \tau_d} \tag{1}$$

Applying this equation with the constants taken from our system we multiply the data with this discriminant to get accurate values for frequency fluctuations in our desired unit of Hz/V.

2. **Uncertainty in Signal**

The consideration of uncertainty in the measured signal is imperative to the analysis of data. It gives us an understanding of how precise or imprecise the readings taken are. Typically the coherence of a signal is used to give us the uncertainty of a signal. Coherence is a statistic that characterizes the relation between two signals, typically the power transfer between the input and output of a system. Due to technical limitations, the coherence could not be measured. Instead, we estimate statistical uncertainty by taking the percentile distribution and setting upper and lower bounds to it, specifically a lower bound of 15.865 and an upper bound of 84.135, corresponding to the standard deviation around the mean of a normal distribution.

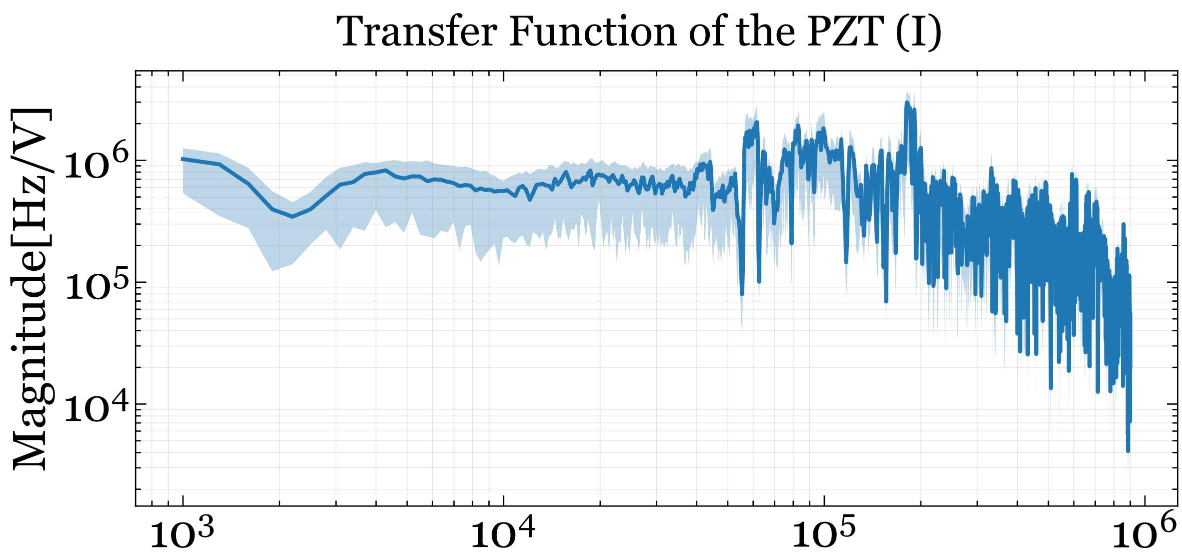

Figure 5: Uncertainty in Signal Magnitude taking Percentile

7 Modelling the Loop

In order to better understand the controls and experiment with them, we model the loop in Python using the control package. This is the auxiliary laser loop which consists of the cavity, PDH sensor, PDH box, and the AUX laser's PZT (referred to as the actuator since it operates the laser). The transfer function of each individual component is found and accordingly their poles, zeroes, and gains. The individual transfer functions are then combined in series to find the resultant transfer function of the system.

When we observe the original transfer function of the loop, it reflects each of the individual components' transfer functions. The cavity pole does not amplify the signal and behaves as a low-pass filter, whose gain at DC is 0. A large amount of gain comes from the uPDH Servo. This gain is increased to get a UGF of 11 kHz. An interesting observation to note here is that the analytical model of the PZT is analogous to that of two spring-coupled mass system under friction. The actuator transfer function has a value of 1e6 at DC.

We model the same loop with the inclusion of a filter that is exactly the inverse of the resonant frequencies caused by the laser PZT. We find that the resonances are canceled in the open loop transfer function as a result. The purpose behind the cancellation is to achieve a phase margin high enough to increase the servo gain. Therefore, the canceling of the resonances implies an improvement in the control system's performance in this frequency range and an overall increase in the loop's bandwidth.

8 Conclusion

The controls of a system directly affect its performance and efficiency, by improving the controls to be more robust the more resilient the system is to its environment and to changes in the plant. The actuator transfer function of the auxiliary laser was measured using the delay line frequency discriminator and an analytical model of the system was created. Accordingly, we inverted the PZT transfer function with the intention of canceling out the peaks in the OLTF introduced by the laser's piezo.

We are trying to remove these resonances from the loop to increase the loop's gain, or particularly Unity Frequency Gain. In doing this, we further improve the system's performance at lower frequencies and improve the detector's sensitivity. Using the analytical model of the loop we have, we can gain an understanding of how adding an inversion to the PZT resonance will affect the performance and stability of our system. With this simulation carried out, we can proceed to actually implement the inverted PZT transfer function using a digital filter.

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