Stochastic Gravitational Wave Background Model Fitting and Detector Consistency Checks

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Abstract

In the search for gravitational waves (GWs), researchers have begun to investigate what makes up the stochastic gravitational wave background (SGWB). Detecting the SGWB may give insight into the decoupling of the gravitational field from the rest of the early evolving Universe and the composition of activity in the Universe at any given time. Researchers are investigating how can we best fit and describe the spectrum of these GW signals. My project will build upon the existing power law spectrum prediction and other functional fittings to find more generic and general models for SGWB. These fits will involve a Gaussian process or spline fitting as an alternative to a power law. This proposal will outline the background, motivations, and plans for this research project for the upcoming summer.

1 Introduction

Gravitational waves are perturbations of the space-time manifold expressed by metric tensor $g_{\mu\nu}$ [6]. We are able to detect these fluctuations of space and time as strain, or change in length per unit length. This strain is detected by ground-based interferometers, two of such detectors are LIGO Hanford, WA and LIGO Livingston, LA. This style of Interferometers involve two beams of light at the same wavelength propagated orthogonally to each other. These beams reflect off mirrors and coherently return to the source. When a gravitational wave passes, it strains the arms of the detector. This causes the light beams to move out of phase with one another, and so when the beams are recombined the resulting change in the interference pattern is evidence of a passing perturbation of spacetime.

Four primary sources of gravitational waves are coalescing binary systems, pulsars, supernovae, and stochastic gravitational wave backgrounds (SGWB) [1]. We know that coalescing binary systems appear as "chirps" with an uncertainty arising from unknown number density of coalescences. These chirps are the only signals we have detected so far. Pulsars should appear as sine waves due to their periodic emission of gravitational waves. Supernovae are extremely challenging sources to understand since we have yet to detect or observe them, and they do not have a deterministic expected signal. The fourth source, SGWB, encompasses the unresolved gravitational wave sources. These unresolved sources include, for example, fluctuations from just after the Big Bang, as well as unresolved astrophysical sources like compact binary coalescences. "Stochastic" refers to a non-deterministic strain signal, either due to the generation process or detector limitations. We cross-correlate data from different detectors to try to detect the SGWB.

In Section 2, I will discuss our motivations for investigating better fits for our SGWB this coming summer. In Section 3, I will give some background on GW sources, the signal we detect, and various tools we use to analyze this signal. In Section 4, I will outline my objectives for this summer and research project. In Section 5, I detail my direction of approach to this project. In Section 6, I give a work plan that will guide my progress this summer.

2 Motivations

Understanding the SGWB will help researchers probe the Universe earlier than electromagnetic signals currently allow [1]. Electromagnetic signals go back to about 400,000 years after the Big Bang, when scattering of particles decreased enough for photons to travel unimpeded. The SGWB could take us as far as 10^{-32} s after the Big Bang because GWs propagate through spacetime without the risk of scattering off particles [7]. For comparison, Planck Time is 5.39×10^{-44} s after the Big Bang. Thus, resolving the primordial background could help paint a more clear picture of the early Universe [6].

Determining a better and more general fit for the SGWB signal will also help us learn about the background itself [1]. As we add more time to our background detection survey, we will gain more spectra of the SGWB. Comparing and fitting these spectra will be helpful to learn about the signals beyond the recognizable, precise events. As Allen [1] mentions, the more we record the signals on multiple detectors simultaneously, the better we will be able to transition the sources of the SGWB from "unresolved" to "resolved", helping contextualize our fitting parameters.

Additionally, developing better fits will help bound the stochastic background signal. Narrowing down the frequency ranges where the SGWB signal is present will be helpful to deduce the components of this signal [9]. By fitting parameters to the models we develop during this project, we may be able to better constrain where to turn our attention in our GW searches. Since GW detection is a relatively new scientific development, interpreting as much of the data as we have now will only help us better understand what makes up the SGWB.

3 Background

3.1 Power Law Spectrum

Most models for a GWB predict a power-law spectrum, which is given by:

$$\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{ref}} \left(\frac{f}{f_{\mathrm{ref}}} \right)^{\alpha}, \tag{1}$$

where $\Omega_{\mathrm{GW}}(f)$ is the energy density per logarithmic frequency interval used to describe the isotropic stochastic background. This quantity can also be expressed as $\Omega_{\mathrm{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\mathrm{GW}}}{df}$ where ρ_c is the critical density and ρ_{GW} is energy density of gravitational waves in the infinitesimal frequency interval f to $f + df$ [1]. Fig. 1, which is reproduced from Renzini 2022, provides a visualization for the energy densities expected from different sources across the frequency interval [6]. Ω_{ref} is the the amplitude at a reference frequency, f_{ref}. α is the spectral index. Both Ω_{ref} and α are constrained using strain data. Right now, we can fit various parameter combinations for different frequency ranges of our spectrum [4]. Fig. 2 from Sachdev (2020) shows the log-log profile of our proposed GWB [8]. As this is not linear ~~so~~ we cannot perfectly fit a power law to the solid red line, the GWB, which is the residual signal obtained after subtracting out the binary neutron stars (BNS). Fig. 2 is what we expect from third generation detectors. So far, we have tried to fit various regions of the frequency spectrum with individual power laws but aim to fit the entire profile as best as possible.

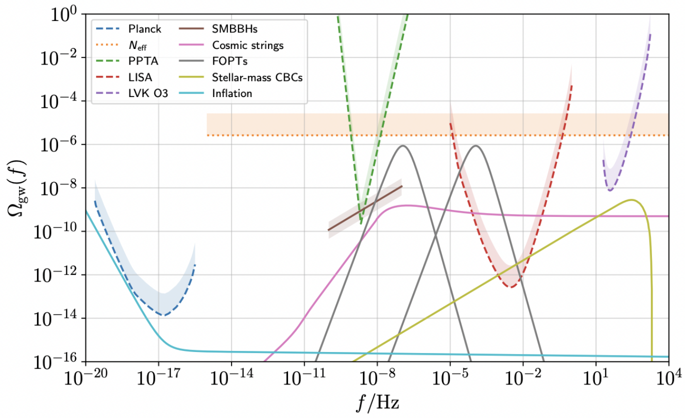

Figure 1: Various GWB sources, their sensitivity, and their energy density with respect to eqn. 1. Figure from Ref [6].

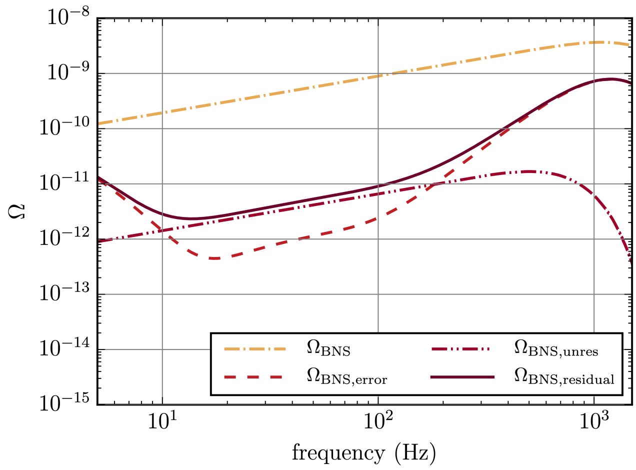

Figure 2: The lines on this log-log plot correspond to our signal and errors. The solid red line is what we expect from a GWB search, which we observe does not resemble a power law after subtracting the binary neutron stars (BNS). Figure from Ref. [8]

3.2 Strain Data Detection & Interpretation

We describe the fluctuations along the arms of the interferometers as strain. This strain, s, can be written as the sum of gravitational-waves, h, and local noise that move the mirrors n [1]:

$$s_i(t) = h_i(t) + n_i(t). \tag{2}$$

We evaluate for $i = 1, 2$, indicating we require two detector signal functions to arrive at a viable strain function. Assuming $h_1(t) \approx h_2(t)$, we can describe the overall signal as:

$$S = \langle s_1(t), s_2(t) \rangle = \int_{-T/2}^{T/2} s_1(t)s_2(t)dt, \tag{3}$$

where T is the detection time. We can use the approximation above to resolve the inner product further into $S \approx \langle h_1, h_2 \rangle + \langle n_1, n_2 \rangle$. This approximation assumes the relative independence of noise and strain. We also generally assume $\langle n_1, n_2 \rangle = 0$ for detectors separated by large distances, such as the detectors in WA and LA. This signal, S, encapsulates the shape of the spectral form we will be searching for starting with the power law and extrapolating to other forms. This direction is explained in depth in Section 5.

3.3 Overlap Reduction Function

Another important ingredient to fitting the SGWB signal is the overlap reduction function (ORF). The overlap reduction function, $\gamma(f)$, reduces the sensitivity from non-parallel alignment of the detector arms and the time delay between the two detectors used. Flanagan (1993) was the first to propose a closed form of the overlap reduction function from the original integral form [3]:

$$\gamma(f) = \frac{5}{8\pi} \int_{S^2} d\hat{\Omega} e^{2\pi i f \hat{\Omega}\cdot\Delta\vec{x}/c} (F_1^+ F_2^+ + F_1^\times F_2^\times), \tag{4}$$

where $\hat{\Omega}$ is the unit vector on S^2, $\Delta\vec{x}$ is the separation between detectors, and $F_i^{+,\times}$ refers to the $+$ or $\times$ polarization of the ith detector. This value falls on a scale of 0 to 1, representing no sensitivity or perfect sensitivity to a specific direction respectively. We can define this polarization in terms of the detector's arms:

$$F_i^{+,\times} = \frac{1}{2} \left(\hat{X}_i^a \hat{X}_i^b - \hat{Y}_i^a \hat{Y}_i^b \right) e_{ab}^{+,\times}(\hat{\Omega}), \tag{5}$$

where $\hat{X}_i^a, \hat{Y}_i^j$ are the directions of the two interferometer arms respectively and $e_{ab}^{+,\times}(\hat{\Omega})$ are the spin-two polarization tensors for the "plus" and "cross" polarizations respectively [1]. $F_i^{+,\times}$ assumes there is no correlation between the polarization directions of GWs.

The importance of the overlap reduction function arises from assuming the GW strains, h_1 and h_2, are equal at both detectors. The overlap reduction function helps make up for the partial overlap of gravity strain in the detectors, helping equate the signals for later processing and analysis. $\gamma(f)$, as shown in Fig. 3, is nontrivially negative for small frequency signals and converges to $\gamma(f) = 0$ for large frequencies. A closed form of the ORF ~~an~~ is an overall emergent function from our data and helps us translate between our data and the signal by averaging over the signal itself.

4 Objectives

The overarching goal of this project is to find a more flexible fitting method for SGWB signals. This main objective divides into a few sub-objectives:

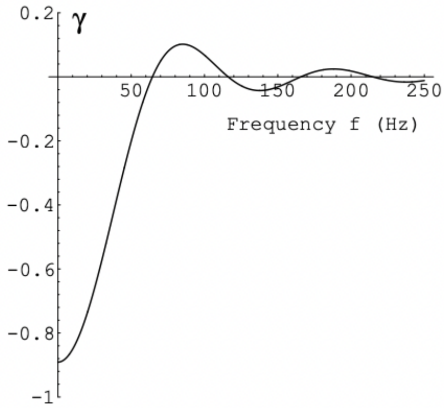

Figure 3: Overlap reduction function between Hanford, WA and Livingston, LA. As f$\rightarrow\ \infty$, $\gamma(f)\ \rightarrow\ 0$. The negative $\gamma(f)$ for low frequencies corresponds to a 90 deg rotation between the two detectors' arms. Figure from Ref. [1].

1. Formulate a fitting relation that optimizes the likelihood function and accurately describes the SGWB spectrum.
2. Develop a fitting relation with generality to describe the range of signal sources included within the SGWB.
3. Standardize consistency checks across detectors to verify signals across multiple interferometers with varying environmental influences and noise.

Non-power law fits have been proposed for the spectra we expect to detect. We also expect the background to no longer resemble a power law after subtracting out foreground events [8] [10]. Furthermore, with 3G detectors, the models could be sensitive enough that they no longer resemble power laws. By developing a general model, we can prepare for this and better understand our ~~second run~~ data [5]. We already observe turnover at higher frequencies, thus increasing our bandwidth to being sensitive to that through our model will be helpful for future investigation [2].

5 Approach

5.1 Alternative Fittings

Current alternative functional fittings for the SGWB are as follows:

Power Law	$\Omega_{\rm GW}(f) = \Omega_{\rm ref}\,(f/f_{\rm ref})^{\alpha}$
Broken Power Law (BPL)	$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm peak}(f/f_{\rm peak})^{\alpha_1} & \text{for } f \leq f_{\rm peak} \\ \Omega_{\rm peak}(f/f_{\rm peak})^{\alpha_2} & \text{for } f > f_{\rm peak} \end{cases}$
Smooth BPL	$\Omega_{\rm GW}(f) = \Omega_{\rm peak}(f/f_{\rm peak})^{\alpha_1}[1 + (f/f_{\rm peak})^{\Delta}]^{(\alpha_2-\alpha_1)/\Delta}$
Triple BPL	$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm peak}(f/f^{(1)}_{\rm peak})^{\alpha_1} & \text{for } f \leq f^{(1)}_{\rm peak} \\ \Omega_{\rm peak}(f/f^{(1)}_{\rm peak})^{\alpha_2} & \text{for } f^{(1)}_{\rm peak} < f \leq f^{(2)}_{\rm peak} \\ k\Omega_{rmpeak}(f/f^{(2)}_{\rm peak})^{\alpha_3} & \text{for } f > f^{(2)}_{\rm peak} \end{cases}$

These models, although simplistic and described by few parameters which require fitting, are not as general and generic as we would like [5]. Alternative functional approaches include Gaussian Processes and Spline Fitting. Gaussian processes are a stochastic method of ensuring all combinations of the variables in a model have a multivariate normal distribution. The benefit of this method is reduced computational time while the trade-off is inaccuracy for large volumes of data. Gaussian processes also provide a built-in confidence internal to the fitting. Spline fitting utilizes smooth, piece-wise polynomials of different degree to describe a curve. Parameters come in the form of coefficients of a polynomial expansion:

$$p_j(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n \tag{6}$$

such that the a_i coefficients allow us to fit an n-degree polynomial to the curve segment j. This is advantageous where a single polynomial fit, such as attempting to use a single power law for the entire spectrum, fails. We will start with spline fitting to recover these parameters and their relationships to each other when constructing functional models for our data.

5.2 Likelihood Functions

When probing parameter space to best fit models to data, we utilize a Gaussian likelihood function for a pair of detectors, i, j[5]:

$$\ln p(\hat{C}_{ij}(f)|\Theta_{\mathrm{GW}}) = \frac{1}{2}\sum_f \frac{\left[\hat{C}_{ij}(f) - \Omega_{\mathrm{GW}}(f,\Theta_{\mathrm{GW}})\right]^2}{\sigma_{ij}^2(f)} - \frac{1}{2}\sum_f [2\pi\sigma_{ij}^2(f)]. \tag{7}$$

Θ encapsulates the parameters we are varying to find the best fit to our data. By slightly varying the parameters in small intervals about our starting parameters, we can determine the truly optimized values of the parameters in relation to one another. We will substitute our spectral fittings, be it a power law, Gaussian process, or spline, into this likelihood function to evaluate the overall statistical fit. We then can use visual checks on the plots of the power spectrum to consider the effect of our fits in conjunction with the data.

In eqn. 6, we will plug in our data for the detector into $\hat{C}_{ij}$. This term is the strain from the detector already adjusted with the ORF as discussed in Section 3.3. We will assume Gaussian and time independent properties of our likelihood function and parameters. By the Central Limit Theorem from statistics, as we use large amounts of data and trials, we should approach a normal, or Gaussian, distribution. Thus this is a safe assumption. Furthermore, we can assume time independence of our data. The background should remain fairly stationary over time, thus when we take more runs of the data, we will be able to create more accurate profiles without having to make adjustments for when the signals were taken. Understanding this likelihood function better will be part of the learning I undertake in the first part of the summer on this project.

6 Work Plan

Date	Phase
June 14 - 17	1
June 20 - 14	1
June 27 - July 1	1
July 4 - 8	2
July 11 - 15	2
July 18 - 22	3
July 25 - 29	3
August 1 - 5	3
August 8 - 12	3
August 15 - 19	3

My project will fall into 3 main phases:

1. Familiarization with SGWB content and code pipelines
2. Basic analysis with data and familiarizing with relationships within data
3. Analysis driven by specific interests within the previous basic analysis and data familiarization

A rough timeline is as follows. This timeline will ensure I have established ample background to complete a meaningful and rigorous project this summer. Program due dates will also be met as given during the program.

References

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