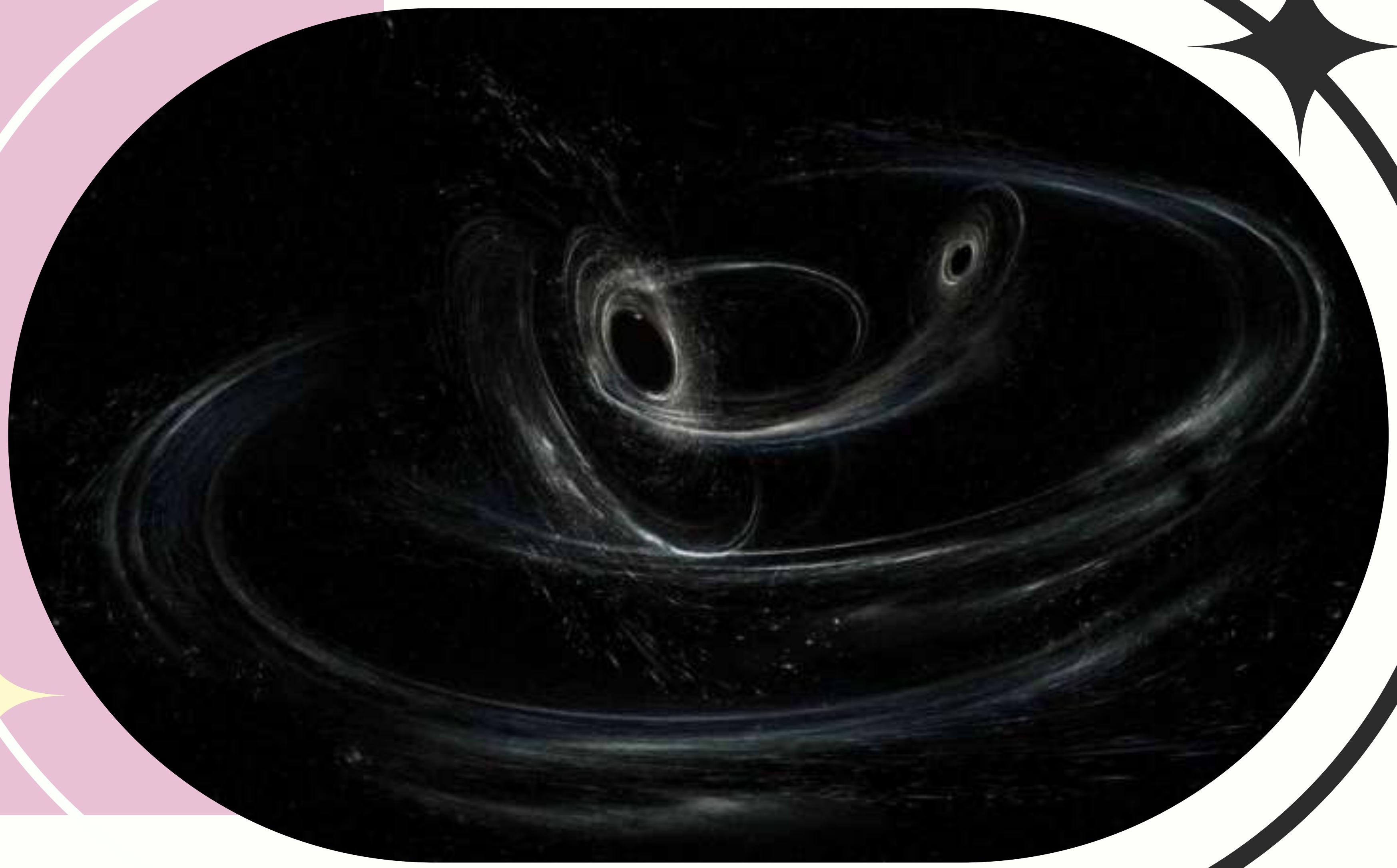


Identifying Correlations in Precessing Gravitational-Wave Signals with Machine Learning

Karen Kang

Mentors: Simona Miller, Katerina Chatziioannou, Deborah Ferguson

01 Background & Motivation

Matched Filtering, NR & Modeling Precessing BBHs

02 Methods

Neural Network & Mapping Algorithm

03 Results

Correlation Recovery & Future Work

01

Motivation & Background

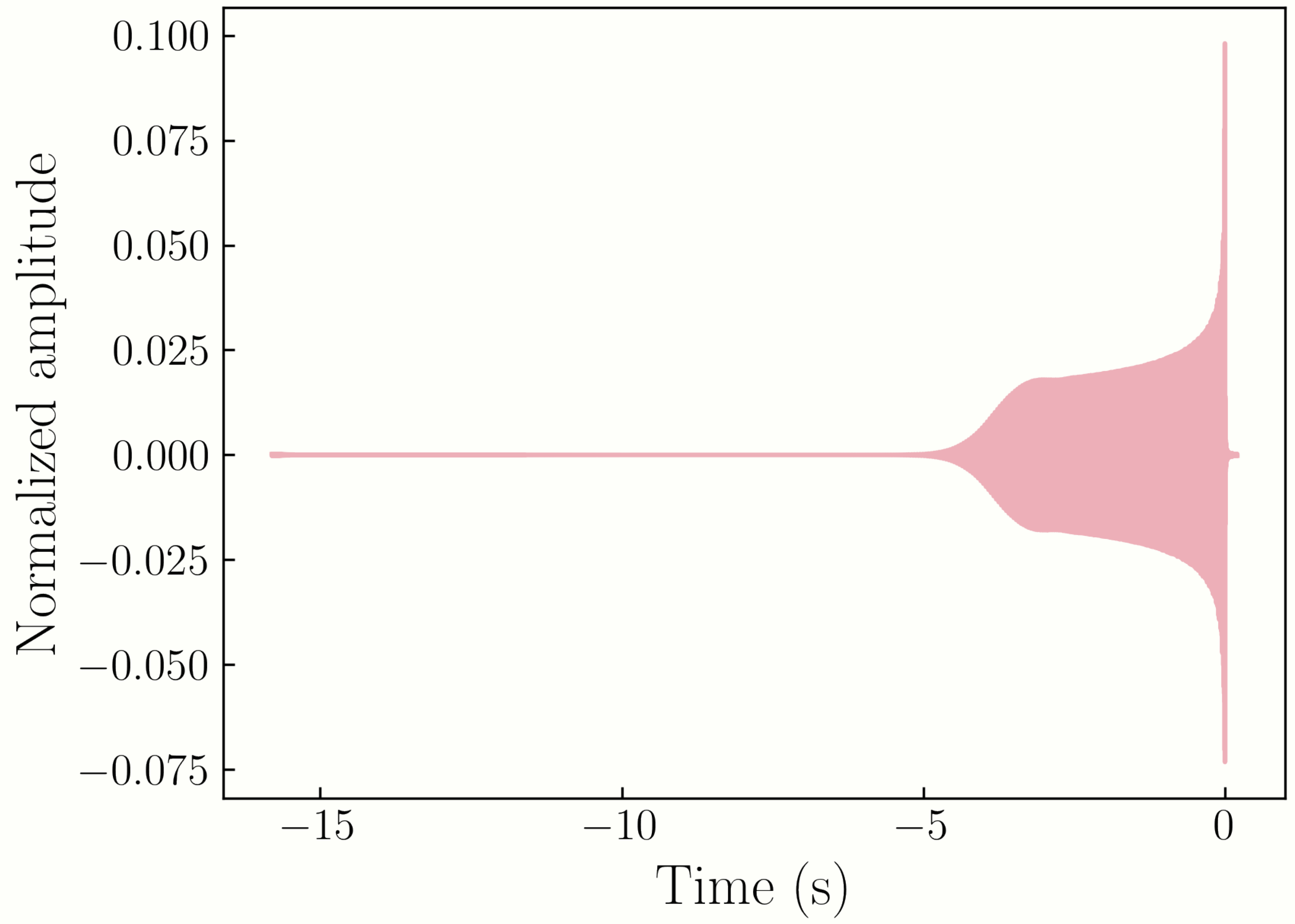
Searching CBCs: Matched Filtering

GW190521: Most Massive BBH Observed!

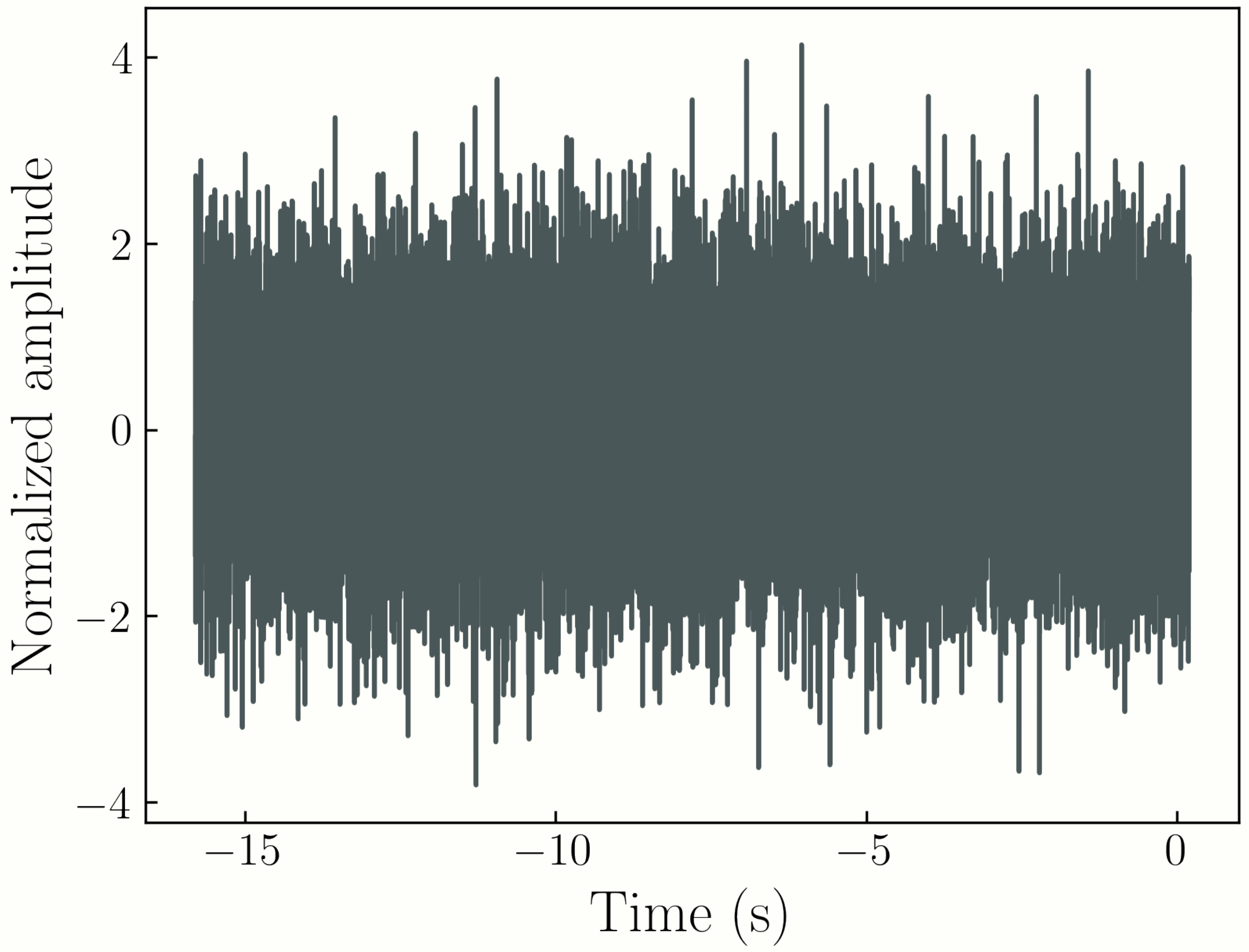
Waveform Correlations: Mass and Precession

Matched-Filtering: Searching for Signal in Detector Data

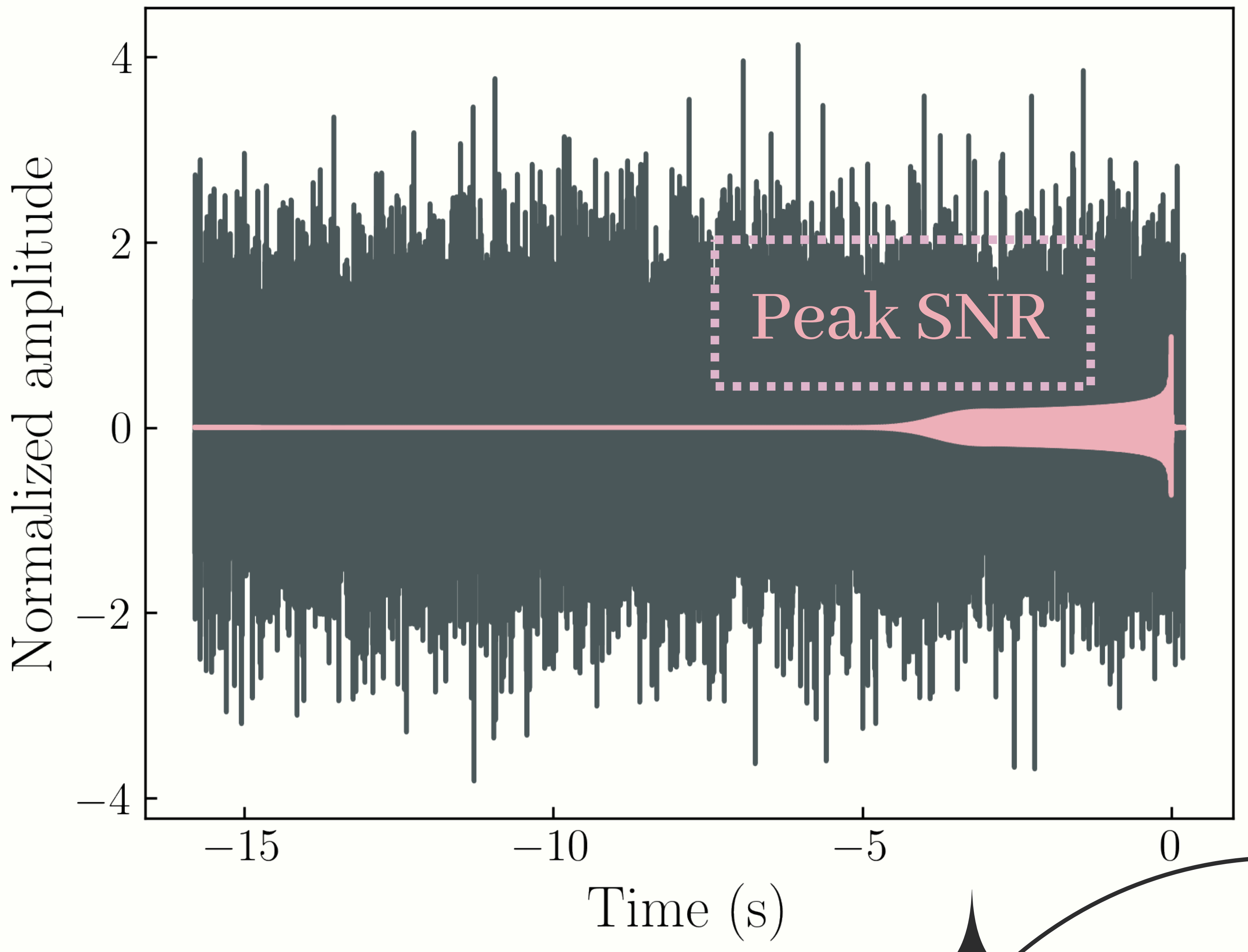
Template



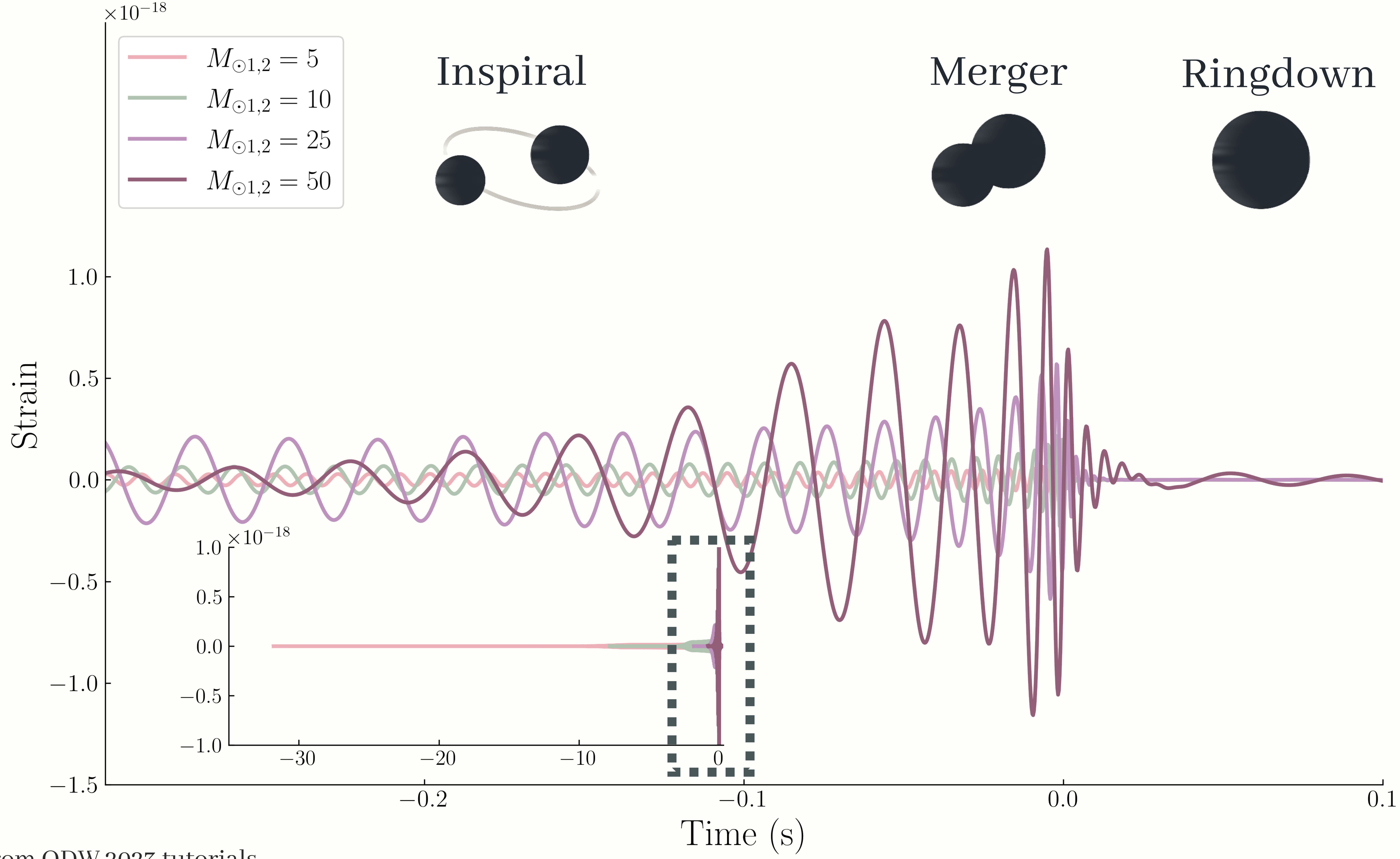
Detected Data



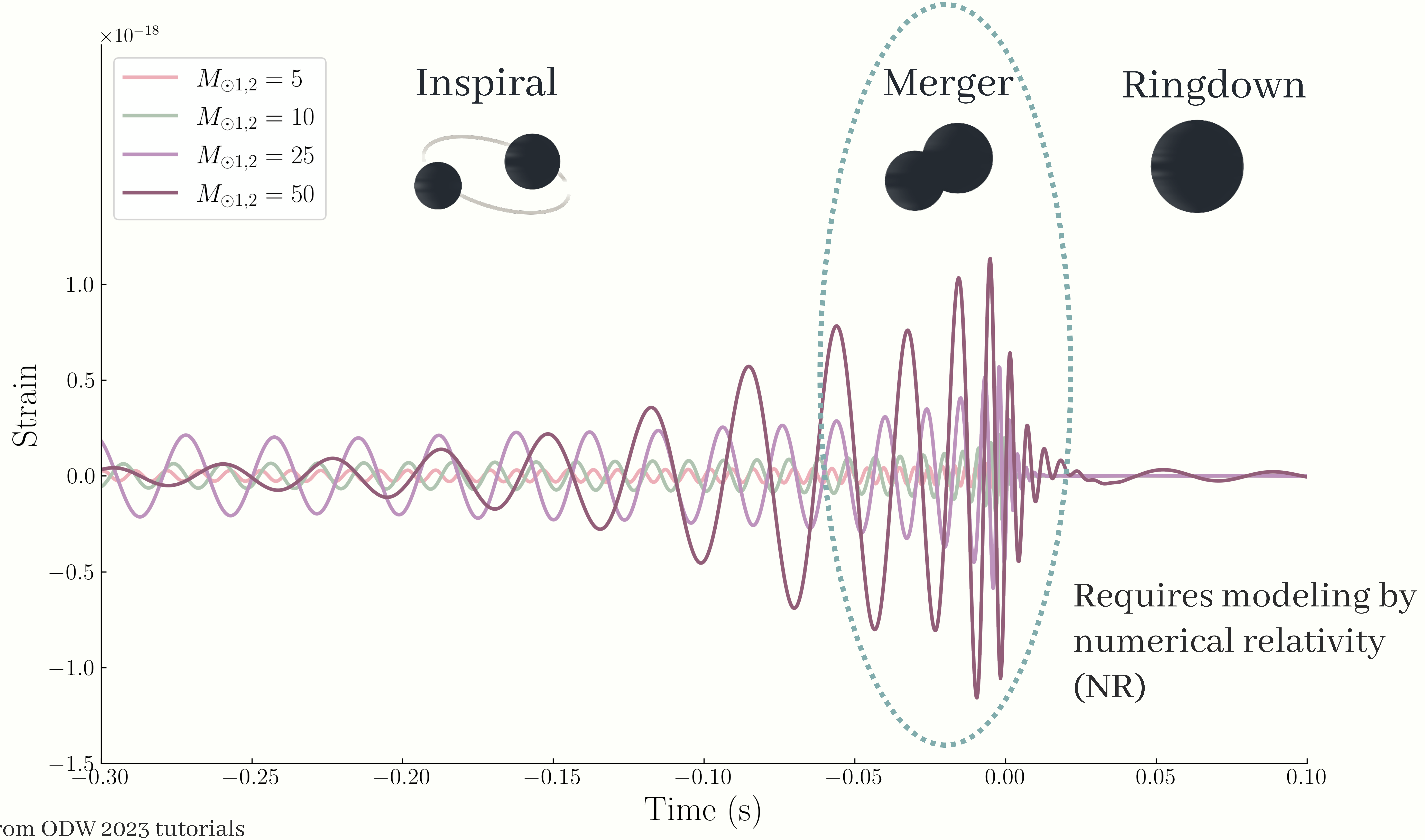
Identify Signal



Adapted from ODW 2023 tutorials

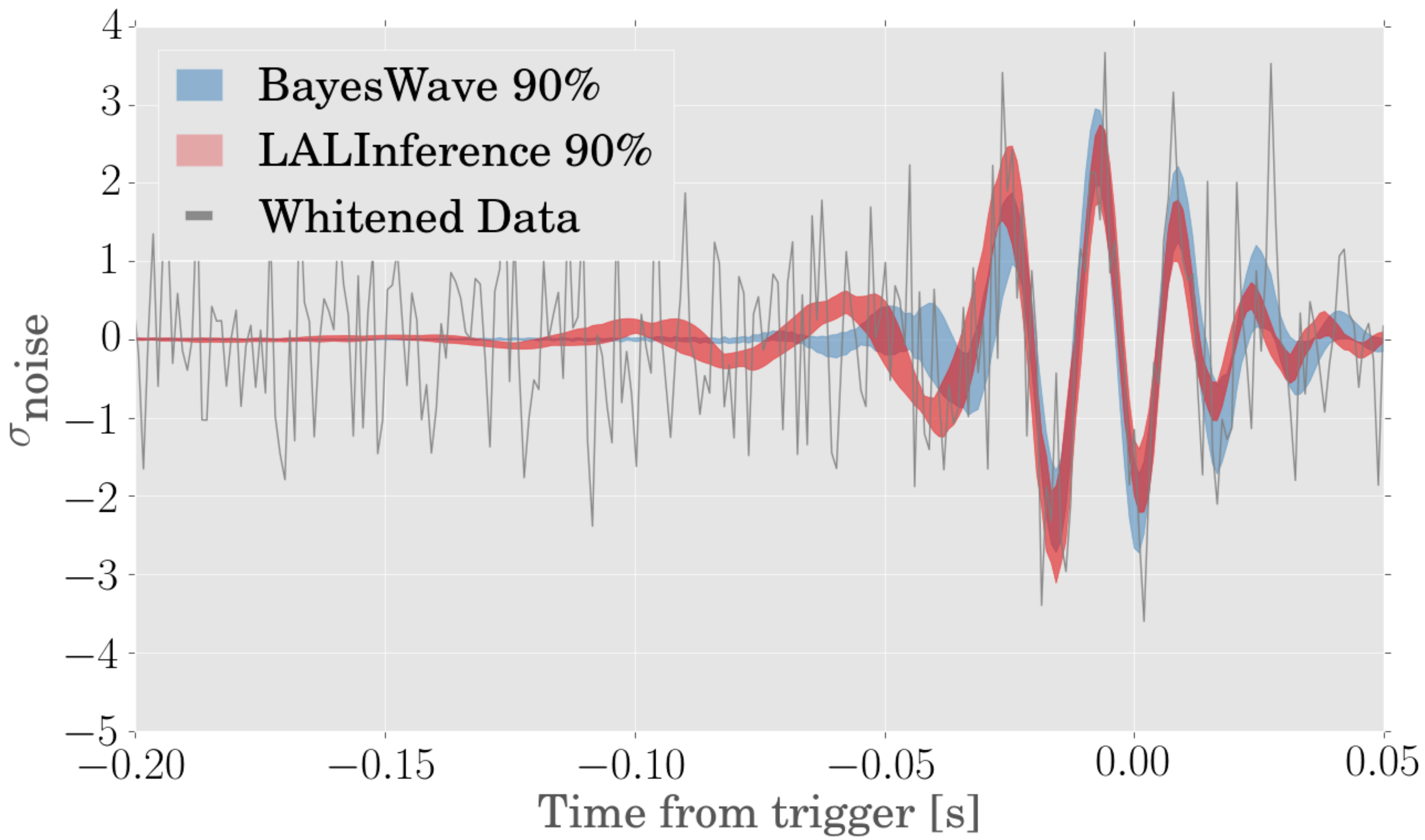


Adapted from ODW 2023 tutorials



Adapted from ODW 2023 tutorials

GW190521 Waveform Reconstruction (LLO)



GraceDB

$M_{Total} = 150 M_\odot$

~5 cycles in the sensitive band

Motivation

Technical

Better informed
- matched-filtering searches
- NR template placement (~months)

Understand the **measurability** of parameters from GW signals

e.g. difficult to estimate parameters when two or more parameters are **degenerate**

Astrophysics

GW190521: heaviest BBH system observed

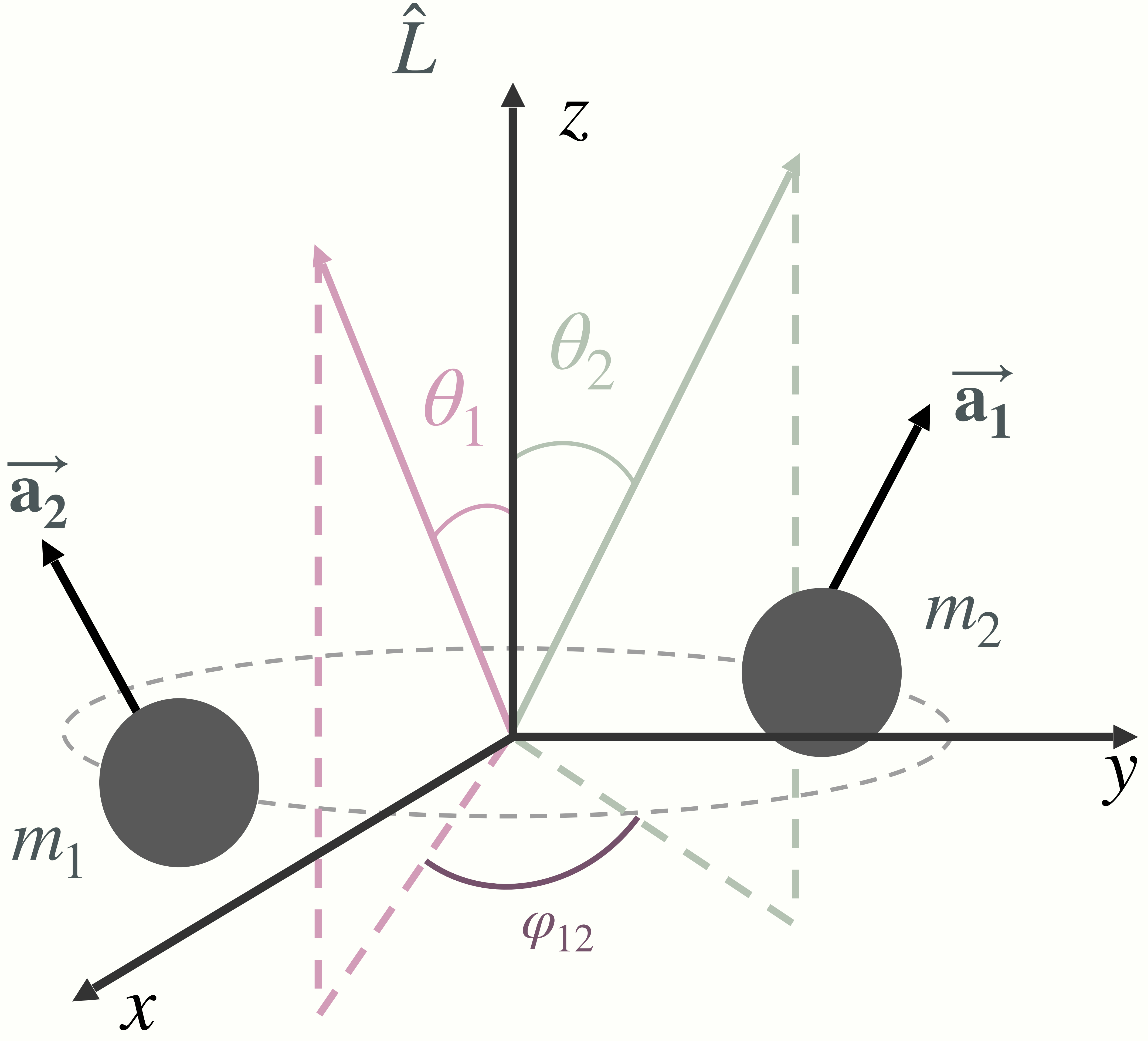
Merger-dominated waveforms

- **Precession effects** remain elusive
- **Spin configurations** indicative of orbital dynamics of progenitor

Intermediate-Mass black boles (IMBHs)

How well can we measure spin parameters of highly massive BBHs from detected GW signals?

Modeling Precessing BBH



- $\hat{L}$ orbital angular momentum direction
- m_1, m_2 component masses
- $\vec{\mathbf{a}}_1, \vec{\mathbf{a}}_2$ spin vectors

  * a_1, a_2 denotes the magnitude [0,1]
- θ_1, θ_2 polar angle between $\hat{L}$ and $\vec{\mathbf{a}}_1, \vec{\mathbf{a}}_2$

Mass Ratios

Mass ratio $$q = \frac{m_1}{m_2} \quad q \geq 1$$

Symmetric mass ratio $$\eta = \frac{q}{\left(q+1\right)^2} \quad 0 \leq \eta \leq 0.25$$

Equal mass

Effective Spin and Precession

Leading order spin effects on inspiral phasing

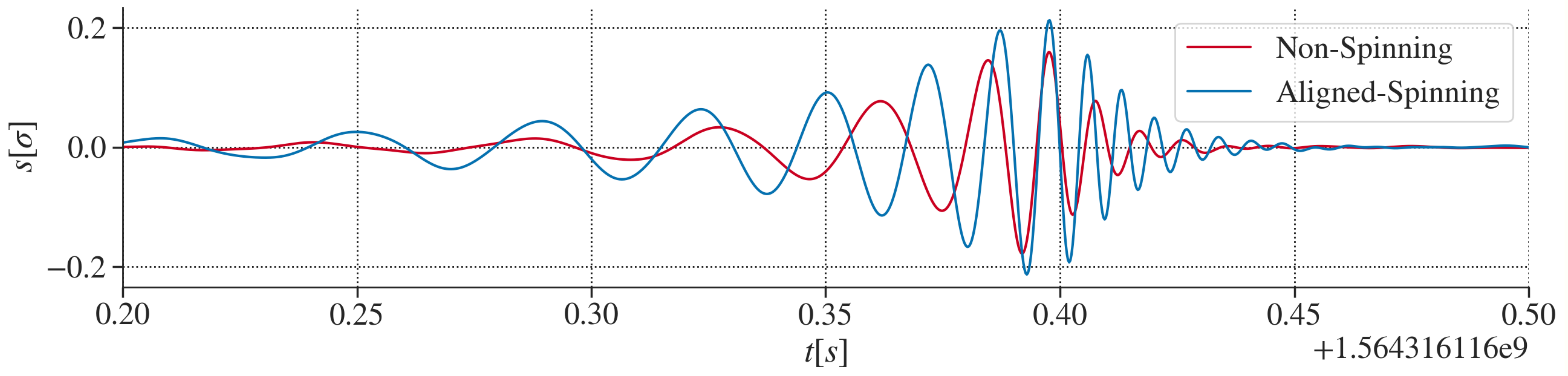
$$\chi_{\text{eff}} = \frac{qa_1 \cos(\theta_1) + a_2 \cos(\theta_2)}{q+1} \qquad -1 \leq \chi_{eff} \leq 1$$

mass-weighted average
of the component spins
aligned with $\hat{L}$

$$\chi_p = \max\left(a_1 \sin\theta_1, \frac{4q+3}{4+3q} qa_2 \sin\theta_2\right) \qquad 0 < \chi_p < 1$$

mass-weighted average
in-plane spin

Effects of Spin on GWs

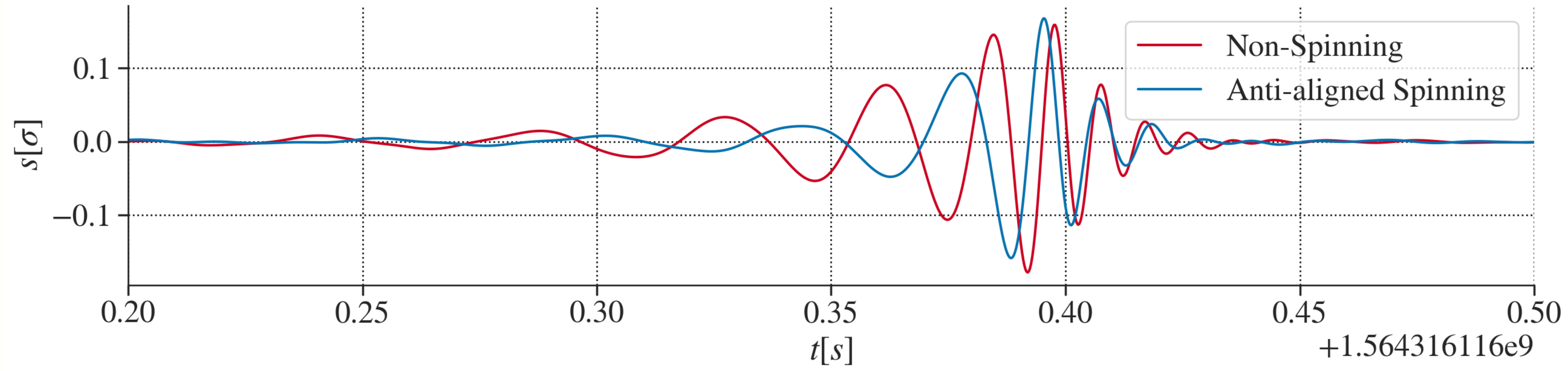


$\chi_{\text{eff}} > 0$

$\vec{a}_1, \vec{a}_2$ aligned with $\hat{L}$

→ BHs inspiral to closer separation

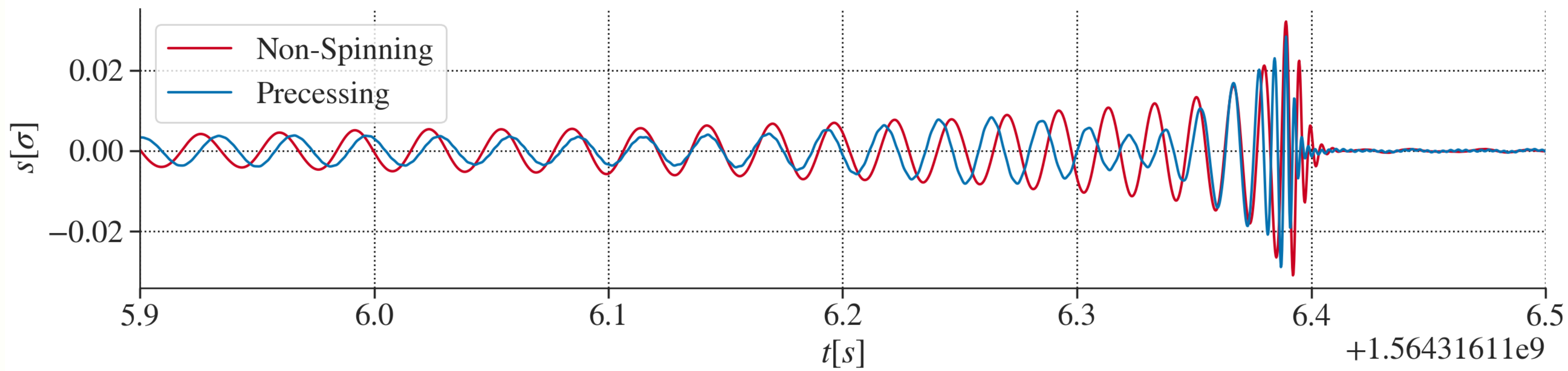
→ longer, stronger GWs



$\chi_{\text{eff}} < 0$

$\vec{a}_1, \vec{a}_2$ anti-aligned with $\hat{L}$

→ shorter, weaker GWs



$\chi_p \neq 0$

$\vec{a}_1, \vec{a}_2$ misaligned with $\hat{L}$

→ orbital precession

→ GWs with modulating amplitude and phase

Adapted from Chandra ODW 2023

02 Methods

- **Mismatch:** metric for waveform degeneracies
- **Predicting mismatches** with machine learning
- **Recipe** for mapping the parameter space

Calculating Waveform Mismatches

$$\mathscr{M}\mathscr{M} = 1 - \max_{t,\phi} \mathscr{O}\left[h_1, h_2\right] \equiv 1 - \max_{t,\phi} \frac{\left\langle h_1 \mid h_2 \right\rangle}{\sqrt{\left\langle h_1 \mid h_1 \right\rangle \left\langle h_2 \mid h_2 \right\rangle}}$$

$$\left\langle h_1 \mid h_2 \right\rangle = 2 \int_{f_0}^{\infty} \frac{h_1^* h_2 + h_1 h_2^*}{S_n} df$$

Calculating Waveform Mismatches

$$\mathscr{M}\mathscr{M} = 1 - \max_{t,\phi} \mathscr{O}\left[h_1, h_2\right] \equiv 1 - \max_{t,\phi} \frac{\langle h_1 \mid h_2 \rangle}{\sqrt{\langle h_1 \mid h_1 \rangle \langle h_2 \mid h_2 \rangle}}$$

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Calculating Waveform Mismatches

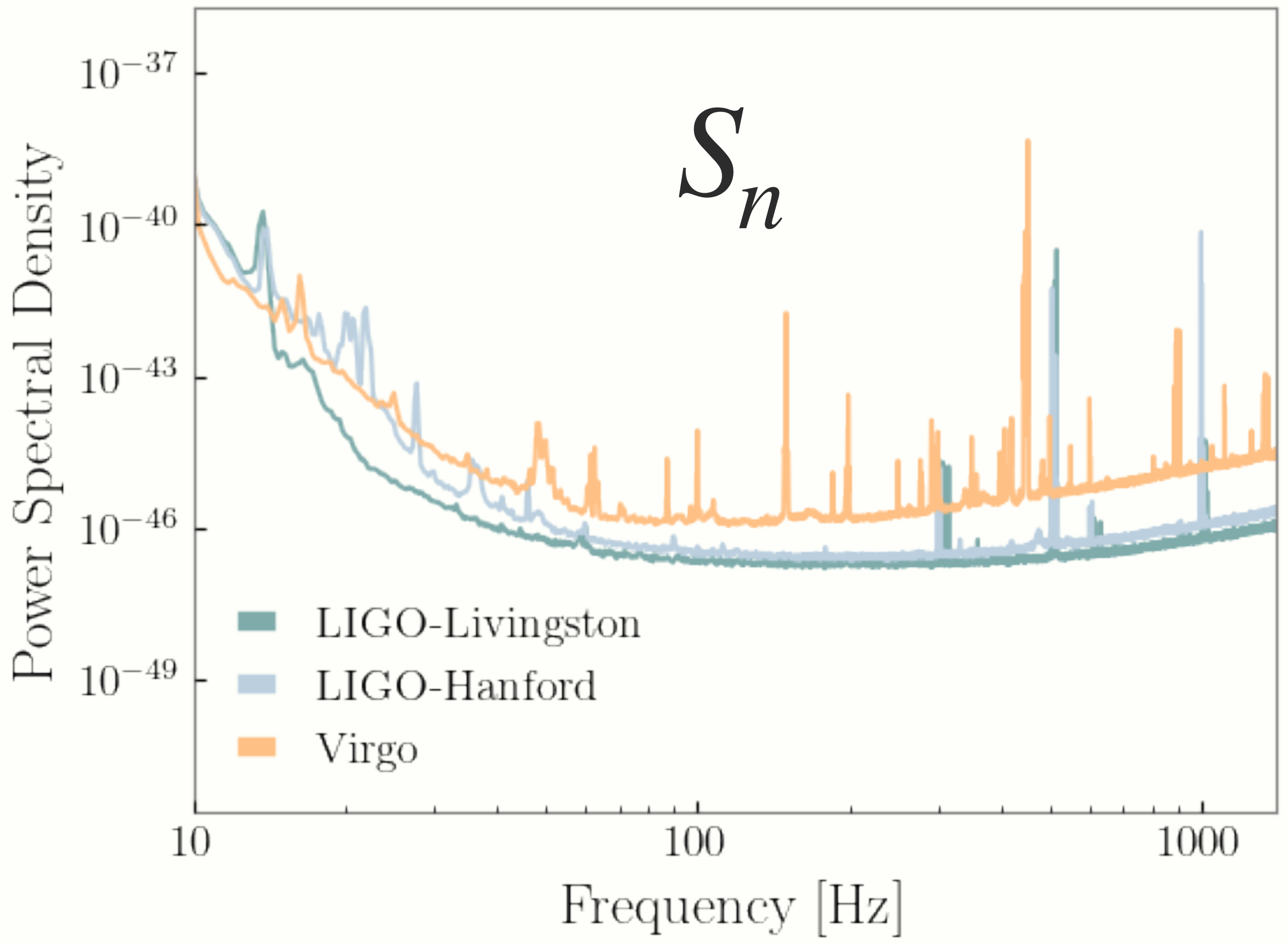
$$\mathcal{M}\mathcal{M} = 1 - \max_{t,\phi} \mathcal{O}\left[h_1, h_2\right] \equiv 1 - \max_{t,\phi} \frac{\langle h_1 \mid h_2 \rangle}{\sqrt{\langle h_1 \mid h_1 \rangle \langle h_2 \mid h_2 \rangle}}$$

Maximize over time and phase of coalescence

Normalized inner product of frequency domain strains

$$\langle h_1 \mid h_2 \rangle = 2 \int_{f_0}^{\infty} \frac{h_1^* h_2 + h_1 h_2^*}{S_n} df$$

One-sided PSD



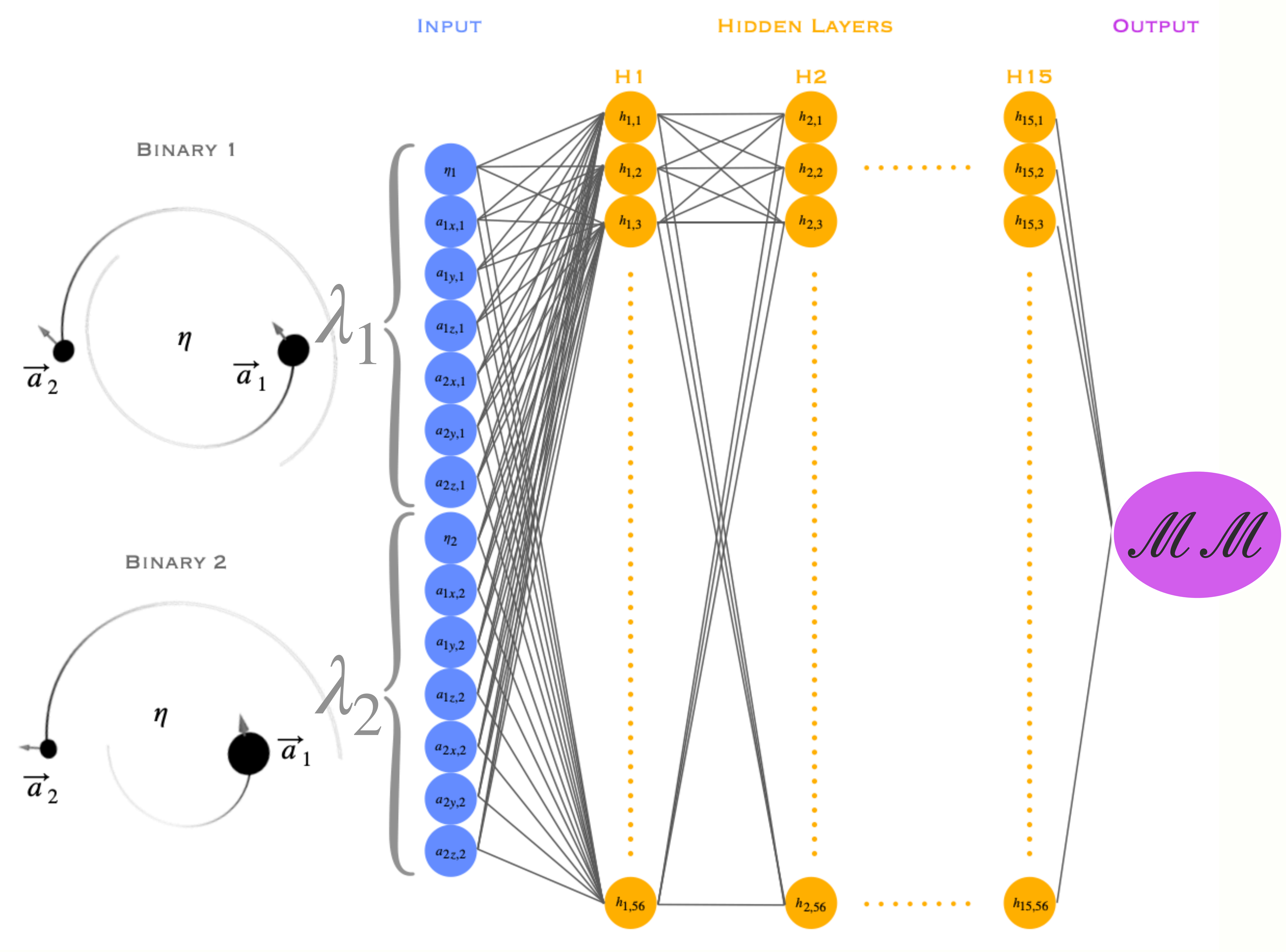
Identical

$$0 \leq \mathcal{M}\mathcal{M} \leq 1$$

Predicting Mismatches Using `mismatch.prediction` Network

$[\eta, a_{1x}, a_{1y}, a_{1z}, a_{2x}, a_{2y}, a_{2z}]$

$\lambda = \eta, \vec{a_1}, \vec{a_2}$

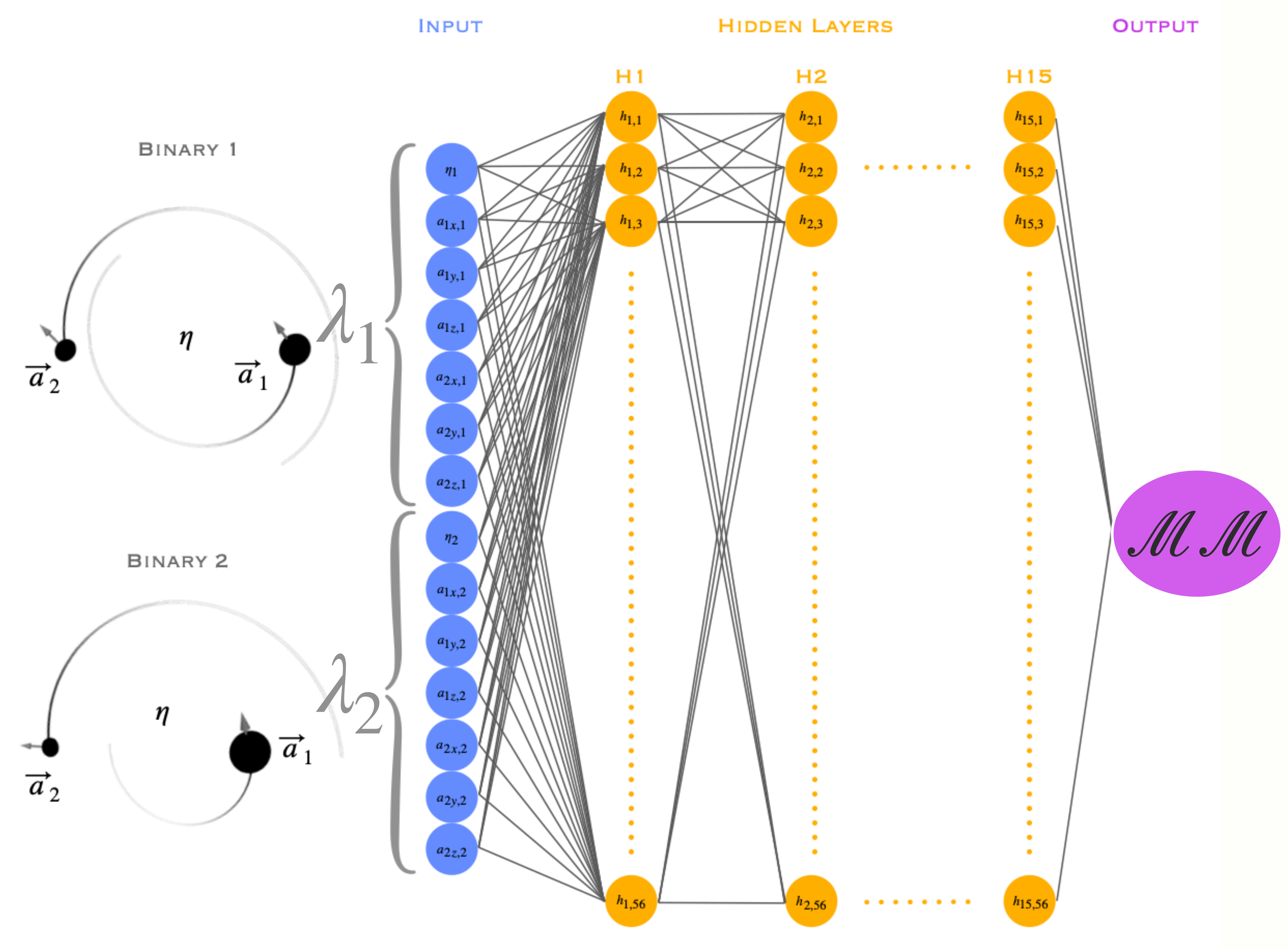


Ferguson (2022)

- Trained on SXS catalog
- Calculates mismatches using $l = 2, m = 2$ modes

Predicting Mismatches Using `mismatch.prediction` Network

$[\eta, a_{1x}, a_{1y}, a_{1z}, a_{2x}, a_{2y}, a_{2z}]$



Ferguson (2022)

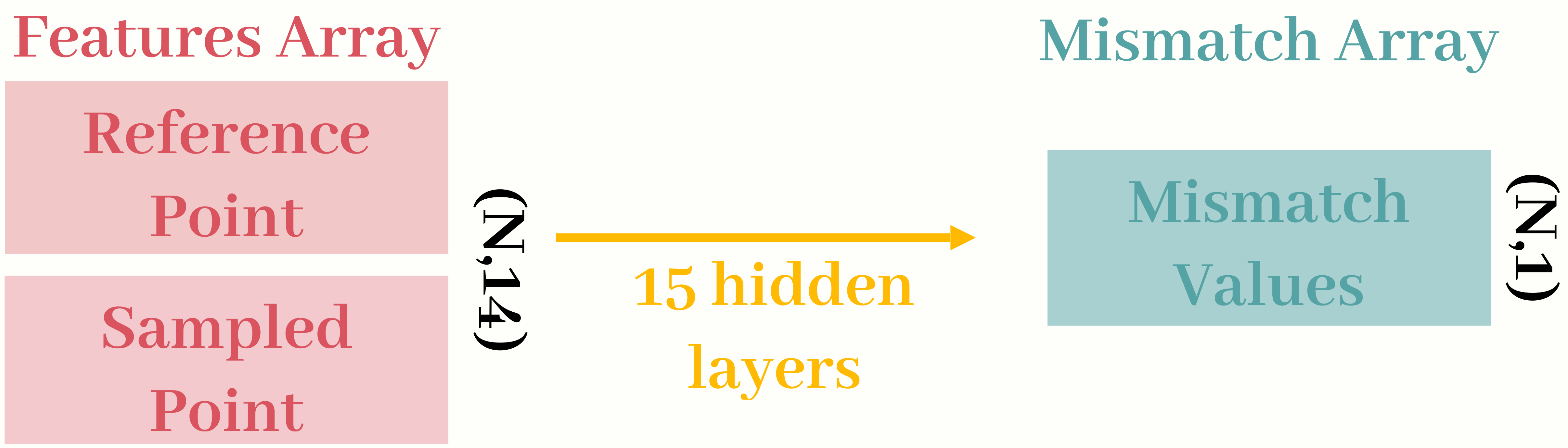
$\lambda = \eta, \mathbf{a_1}, \mathbf{a_2}$

- Trained on SXS catalog
- Calculates mismatches using $l = 2, m = 2$ modes

To study spin parameter space of highly massive, precessing BBHs

- Trained on NRSur7dq4
  Detector frame mass $270 M_\odot$
  Distance 5000 Mpc
- Includes higher order modes (HOMs)

Generating Parameter Space

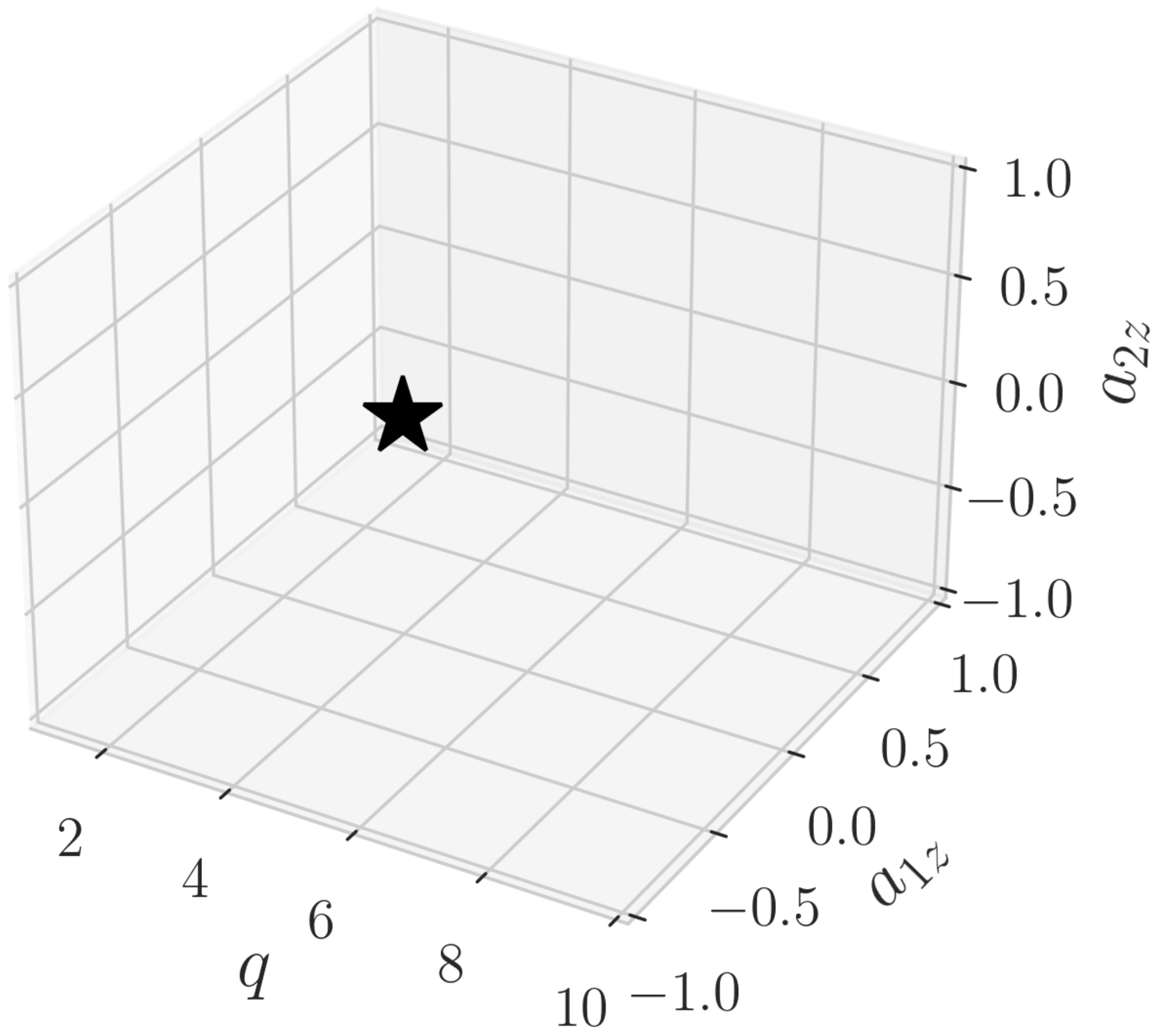


1. **Pick** reference point

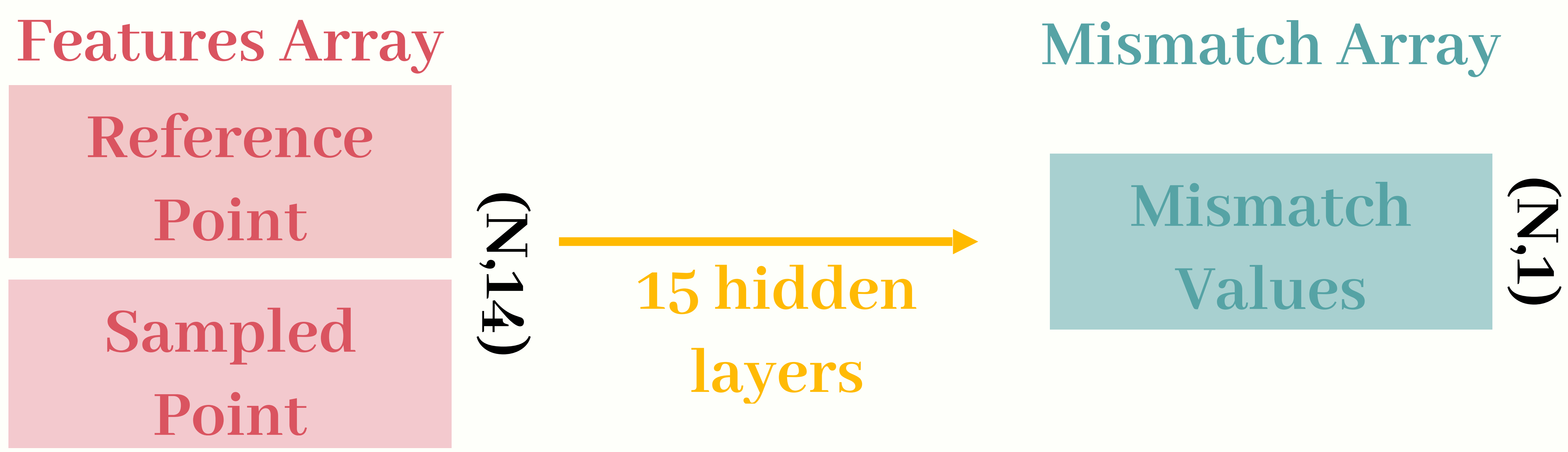
$$\lambda = [0.16,0,0,0,0,0,0]$$

In the q, a_{1z}, a_{2z} space,

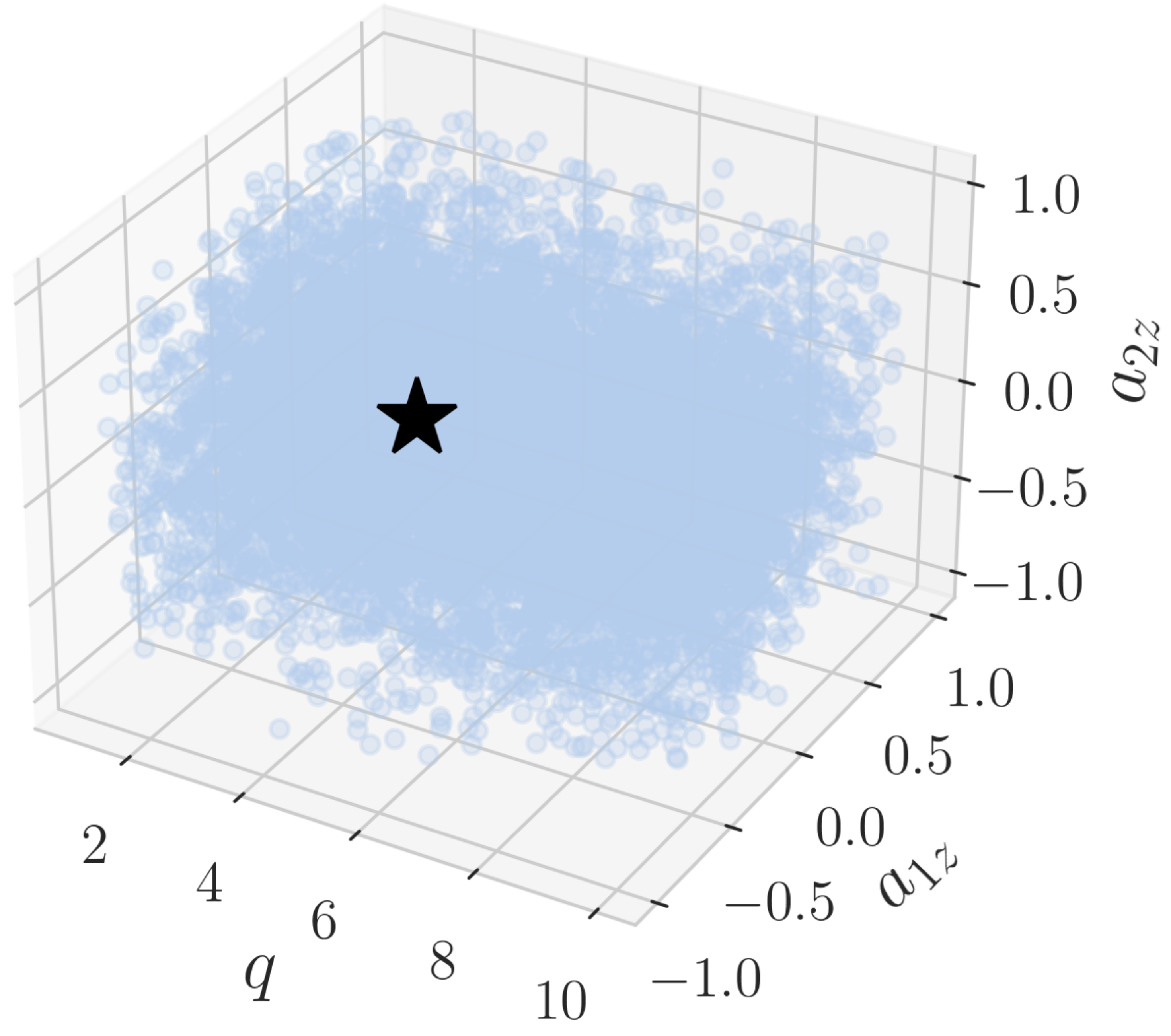
$$x = (4,0,0)$$



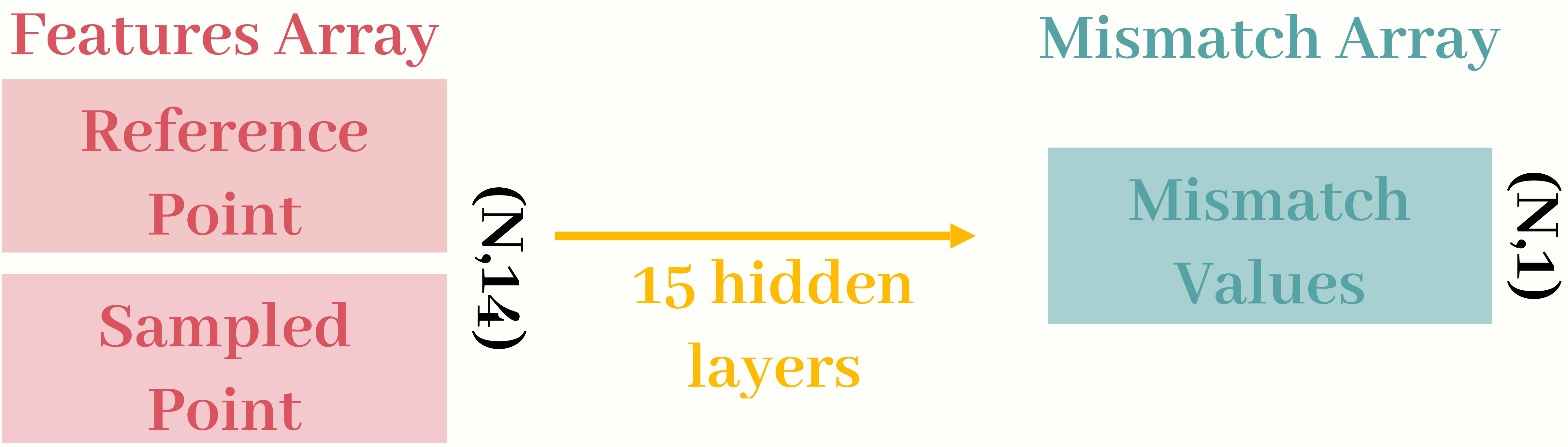
Generating Parameter Space



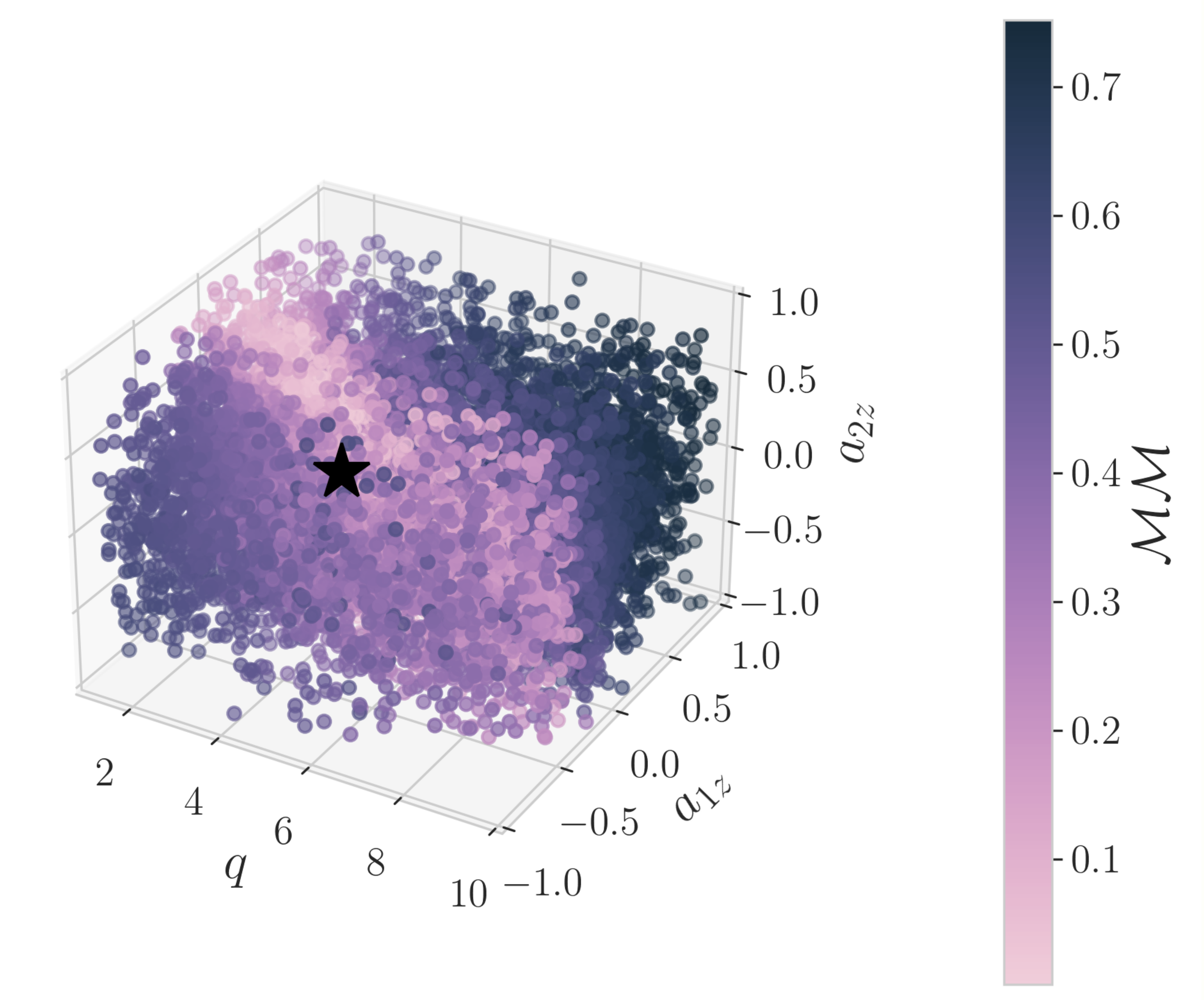
1. Pick reference point
2. **Uniformly sample** points in the parameter space



Generating Parameter Space



1. Pick reference point
2. Uniformly sample points in the parameter space
3. Calculate mismatch of sampled point w.r.t. reference

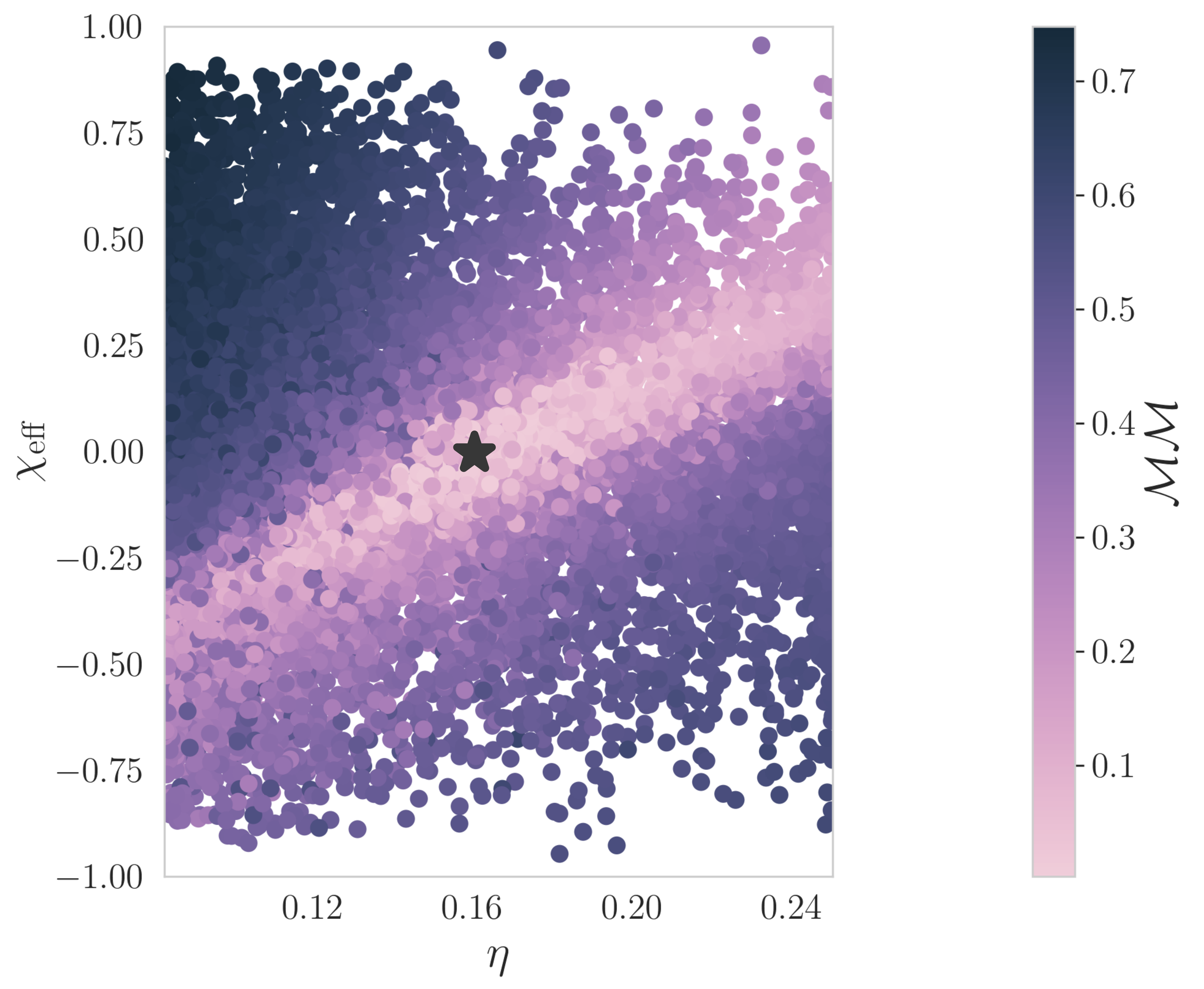


Simple Recipe for Mapping Degeneracies

0. Pick starting injection

1. Generate parameter space

~ 10000 pts

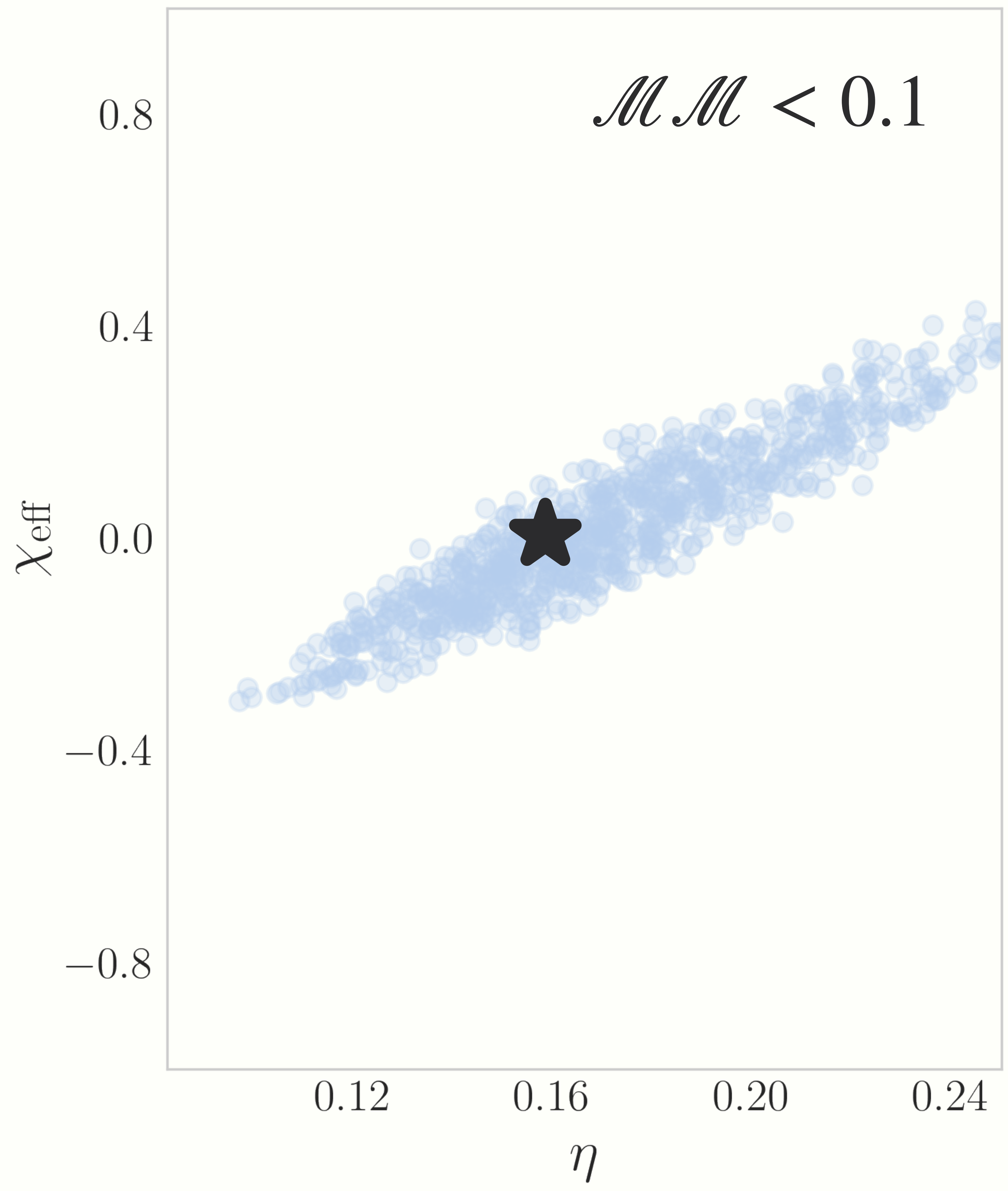


Simple Recipe for Mapping Degeneracies

0. Pick starting injection

1. Generate parameter space

2. Identify degenerate region



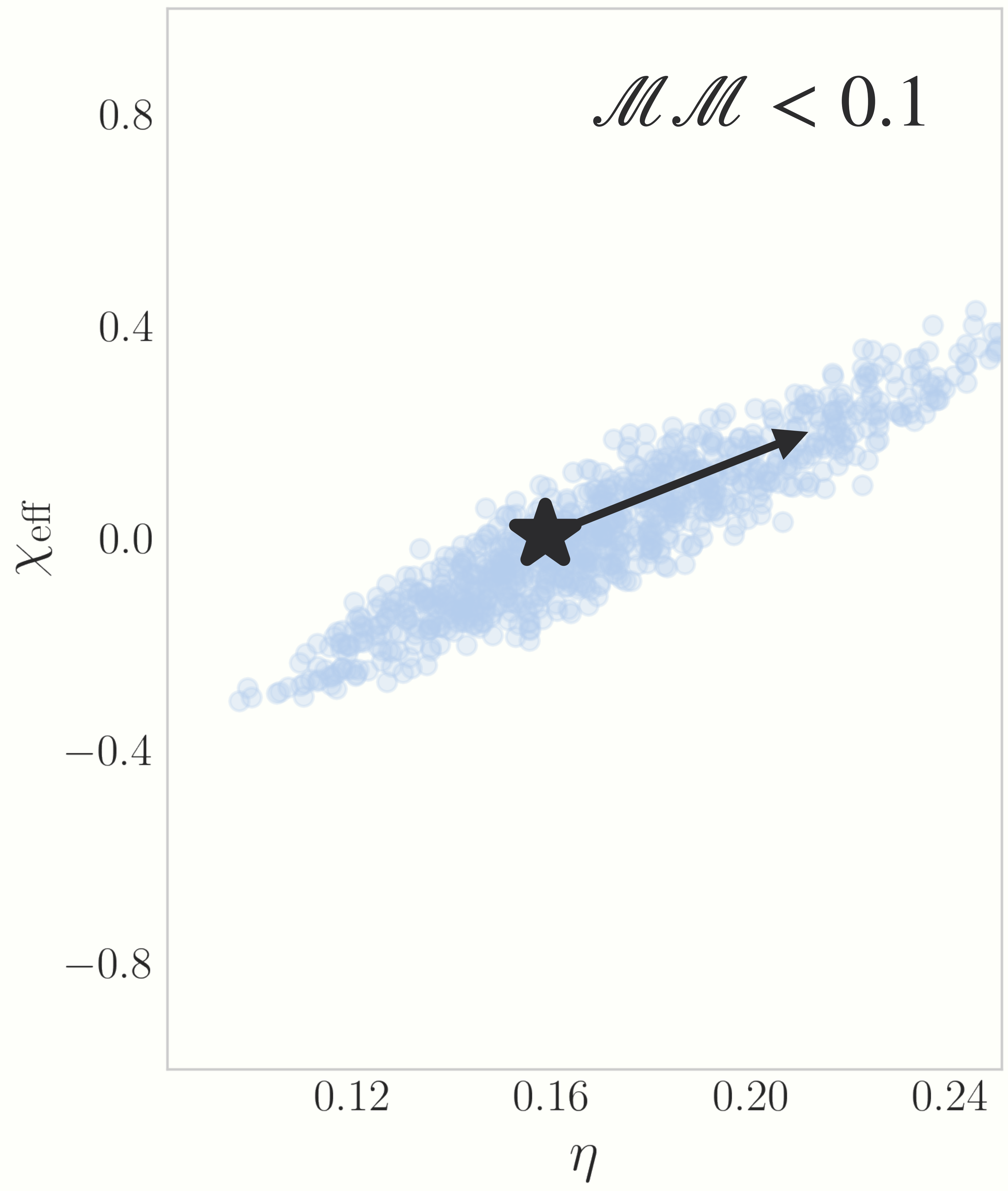
Simple Recipe for Mapping Degeneracies

0. Pick starting injection

1. Generate parameter space

2. Identify degenerate region

3. Find directions to map to (locally)
   1. **Principle Component Analysis (PCA):** principle component with greatest variance
   2. **Bayesian Gaussian Modeling (BGM):** eigenvector of covariance matrix with largest eigenvalue



Simple Recipe for Mapping Degeneracies

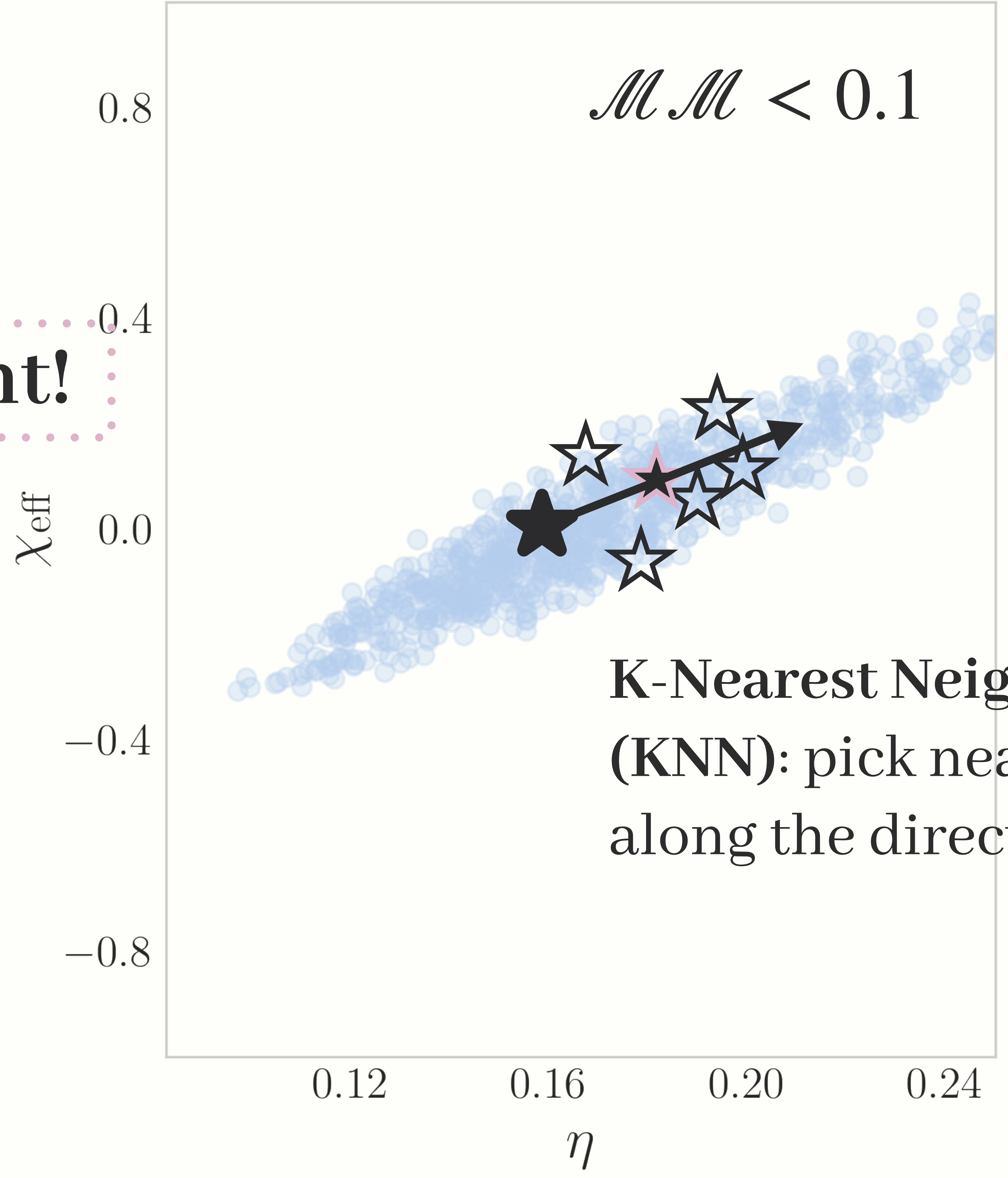
0. Pick starting injection

1. Generate parameter space

2. Identify degenerate region

New point!

3. Find directions to map to (locally)
   1. **Principal Component Analysis (PCA):** principle component with greatest variance
   2. **Bayesian Gaussian Modeling (BGM):** eigenvector of covariance matrix with largest eigenvalue



K-Nearest Neighbor (KNN): pick nearest point along the direction

Simple Recipe for Mapping Degeneracies

0. Pick starting injection

1. Generate parameter space

2. Identify degenerate region

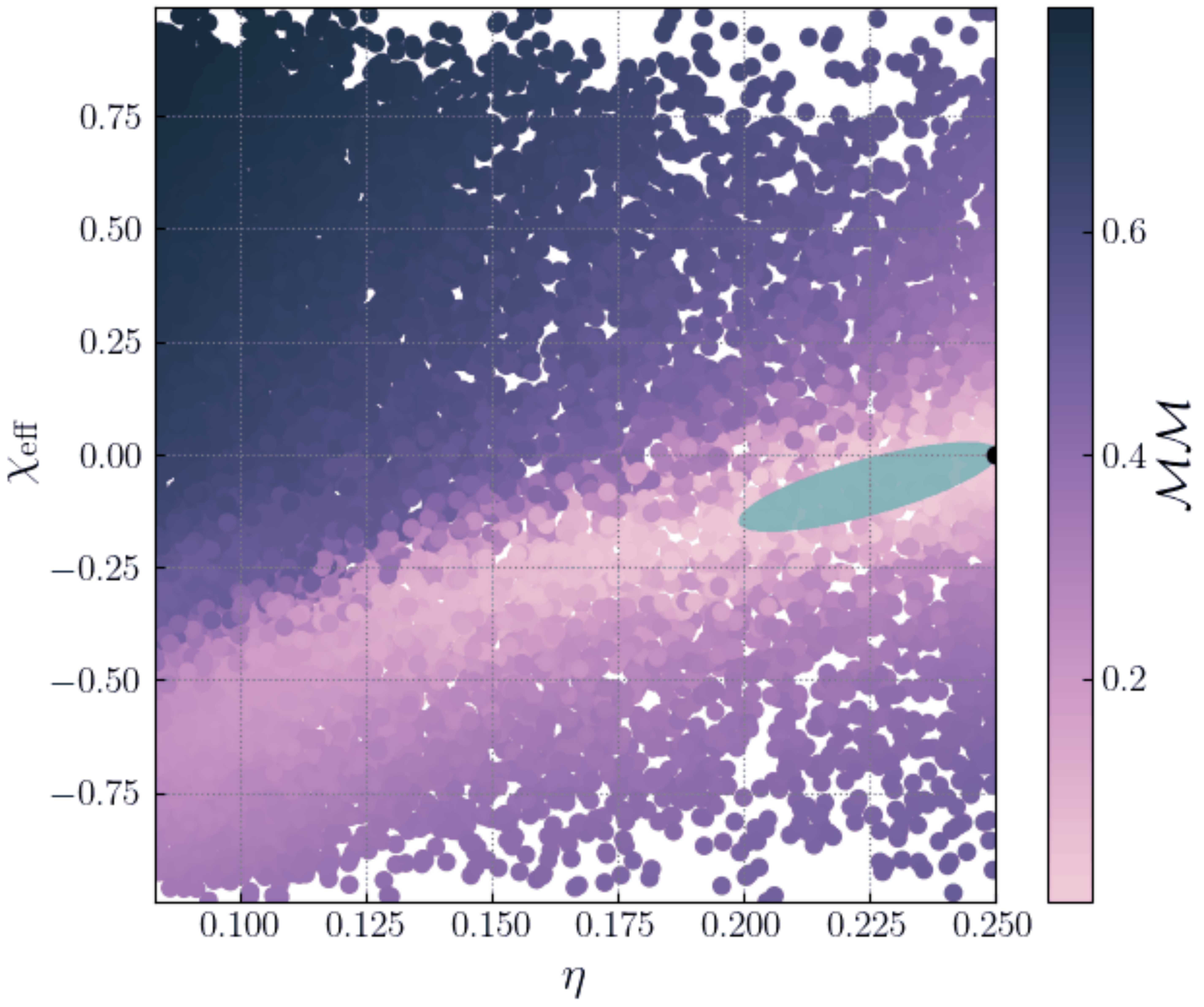
3. Find directions to map to (locally)

New point!

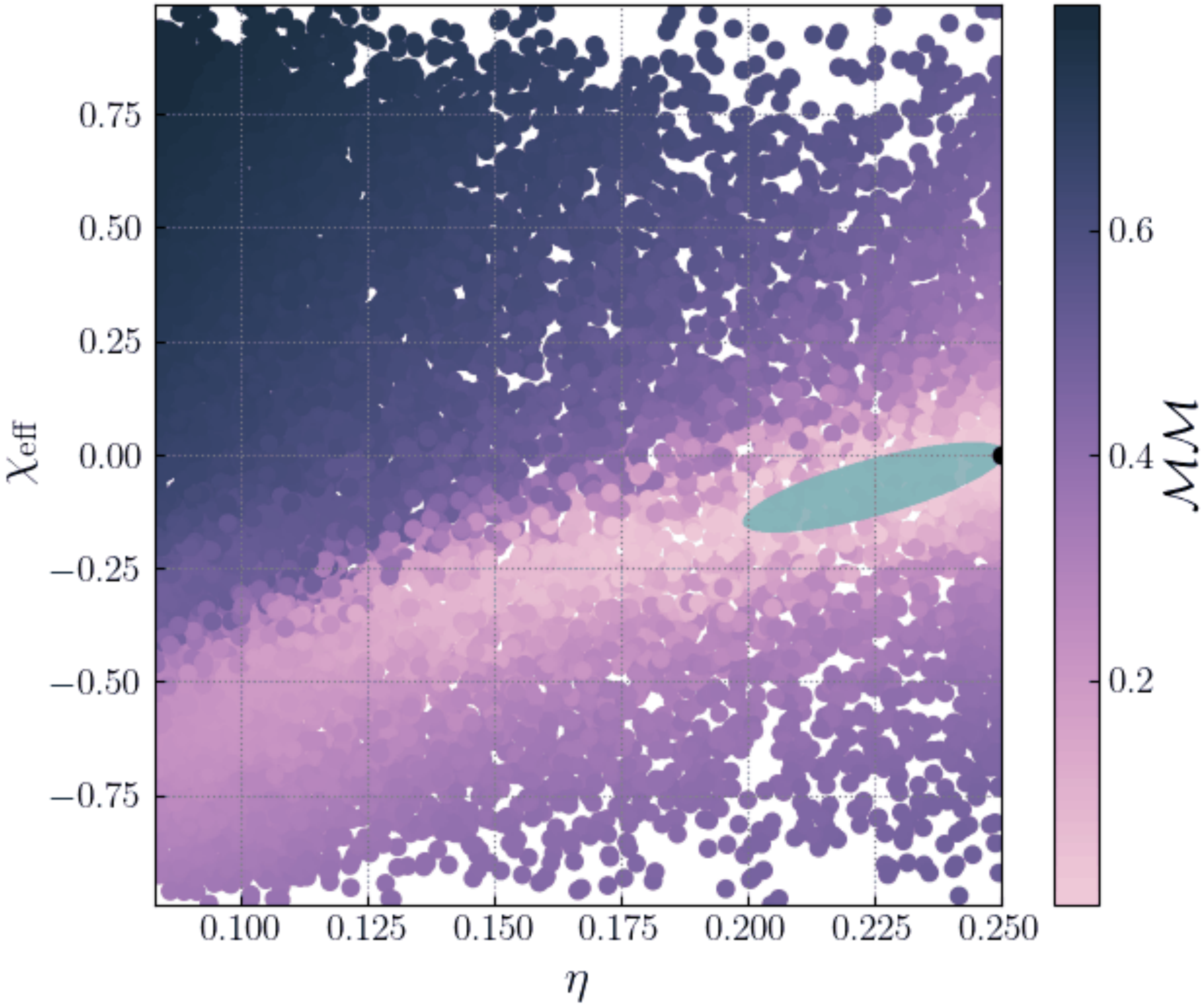
Reached boundary

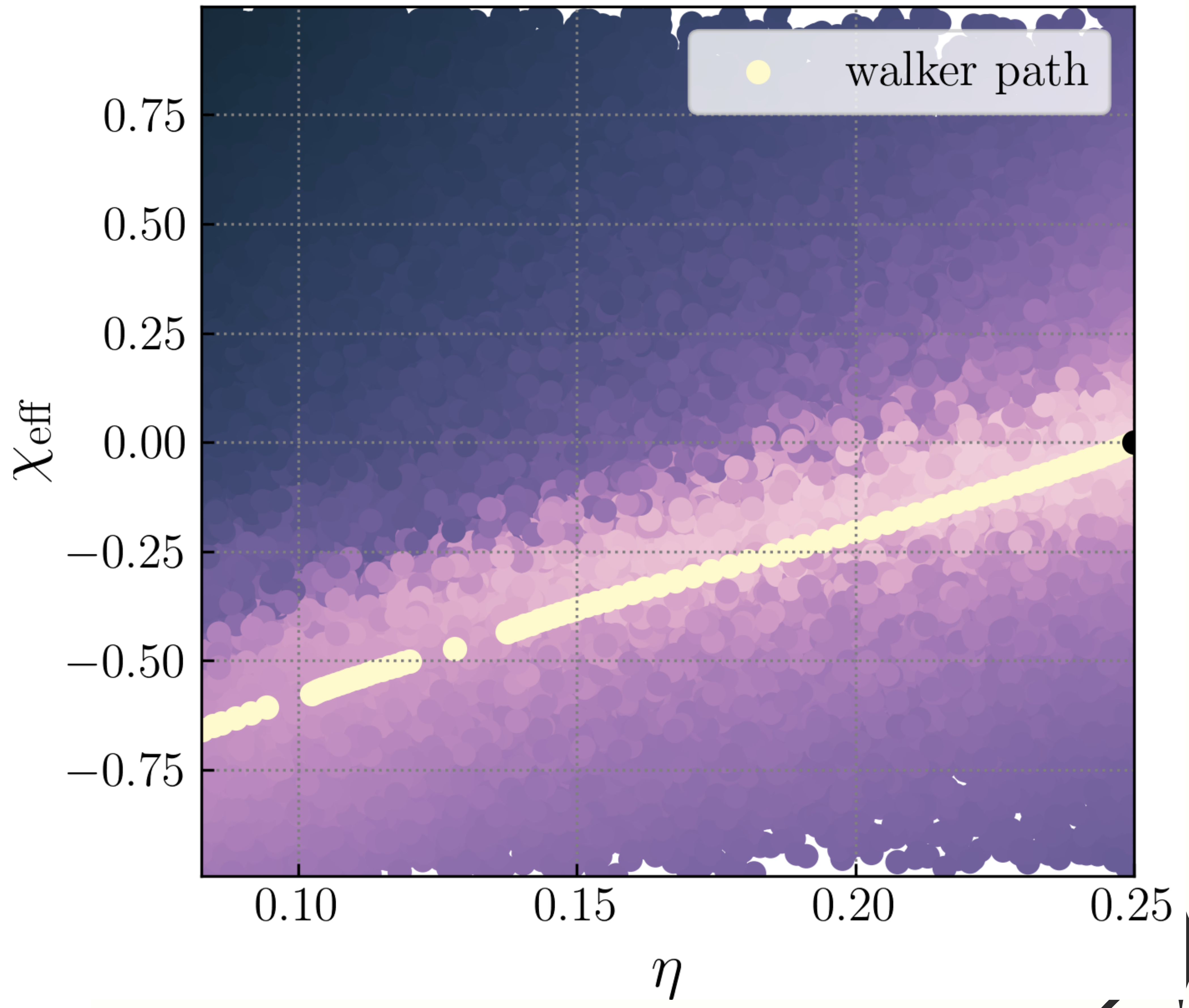
03

Preliminary Results + Future Work



0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
$\chi_{\rm eff}$
0.100
0.125
0.150
0.175
0.200
0.225
0.250
η
0.6
0.4
0.2
$\mathcal{MM}$



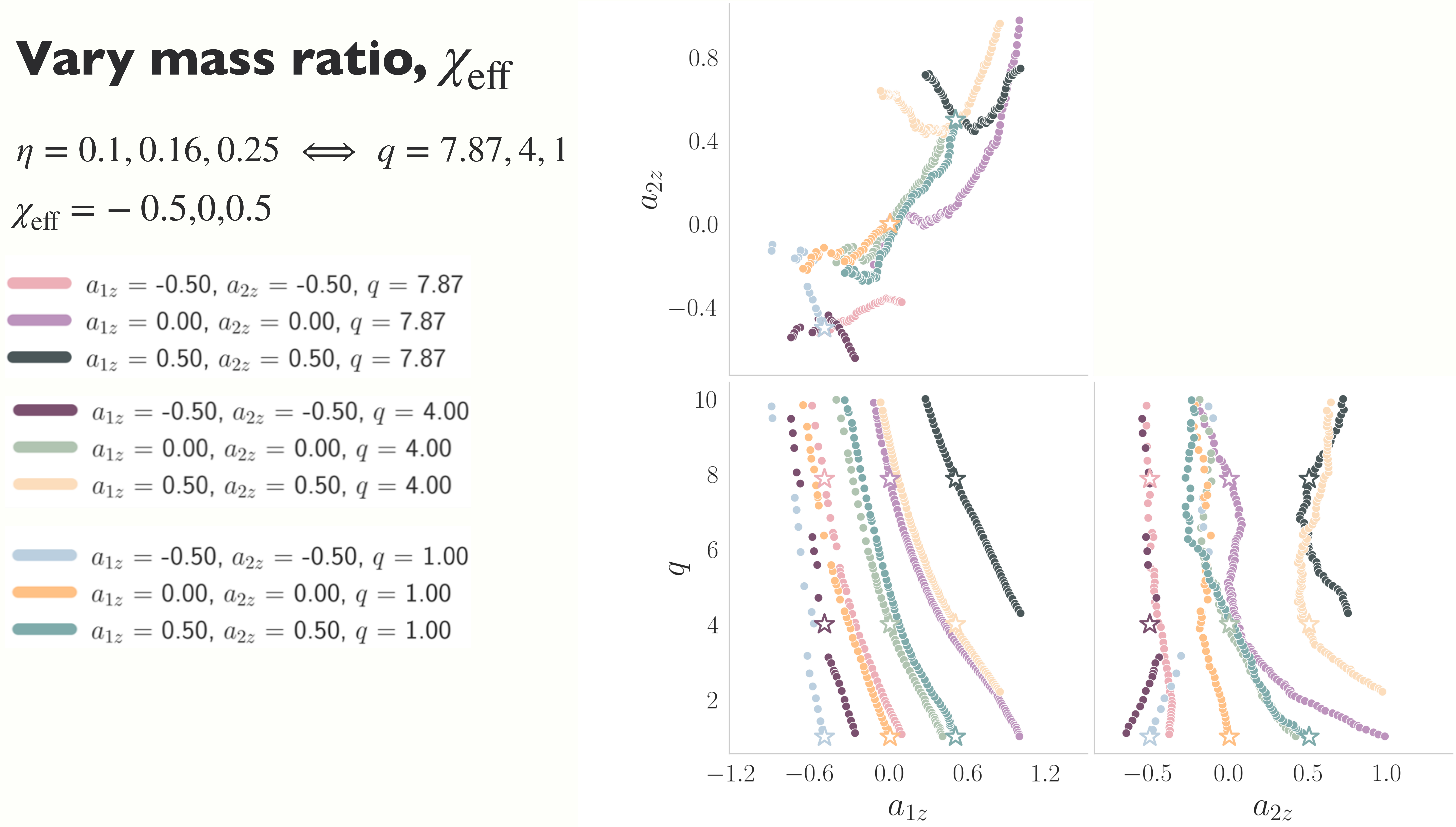


$\lambda = [0.16,0,0,0,0,0,0]$

Vary mass ratio, χ_{eff}

$\eta = 0.1, 0.16, 0.25 \iff q = 7.87, 4, 1$

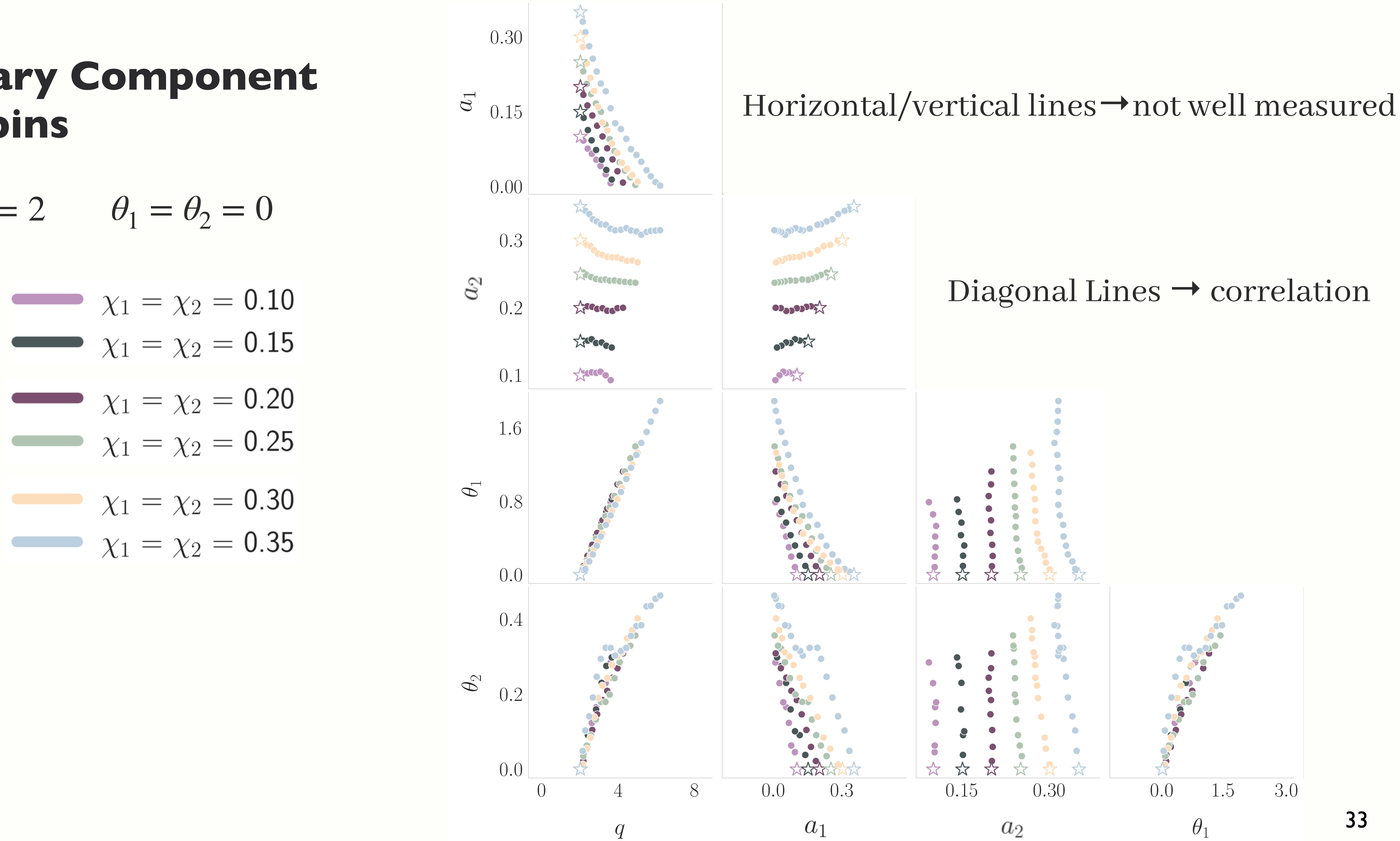
$\chi_{\text{eff}} = -0.5, 0, 0.5$



Starting Injections
a_{1z} = -0.50, a_{2z} = -0.50, q = 7.87
a_{1z} = 0.00, a_{2z} = 0.00, q = 7.87
a_{1z} = 0.50, a_{2z} = 0.50, q = 7.87
a_{1z} = -0.50, a_{2z} = -0.50, q = 4.00
a_{1z} = 0.00, a_{2z} = 0.00, q = 4.00
a_{1z} = 0.50, a_{2z} = 0.50, q = 4.00
a_{1z} = -0.50, a_{2z} = -0.50, q = 1.00
a_{1z} = 0.00, a_{2z} = 0.00, q = 1.00
a_{1z} = 0.50, a_{2z} = 0.50, q = 1.00
a_{2z}
q
a_{1z}
a_{2z}
$\chi_{\rm eff}$
η
$\mathcal{MM}$

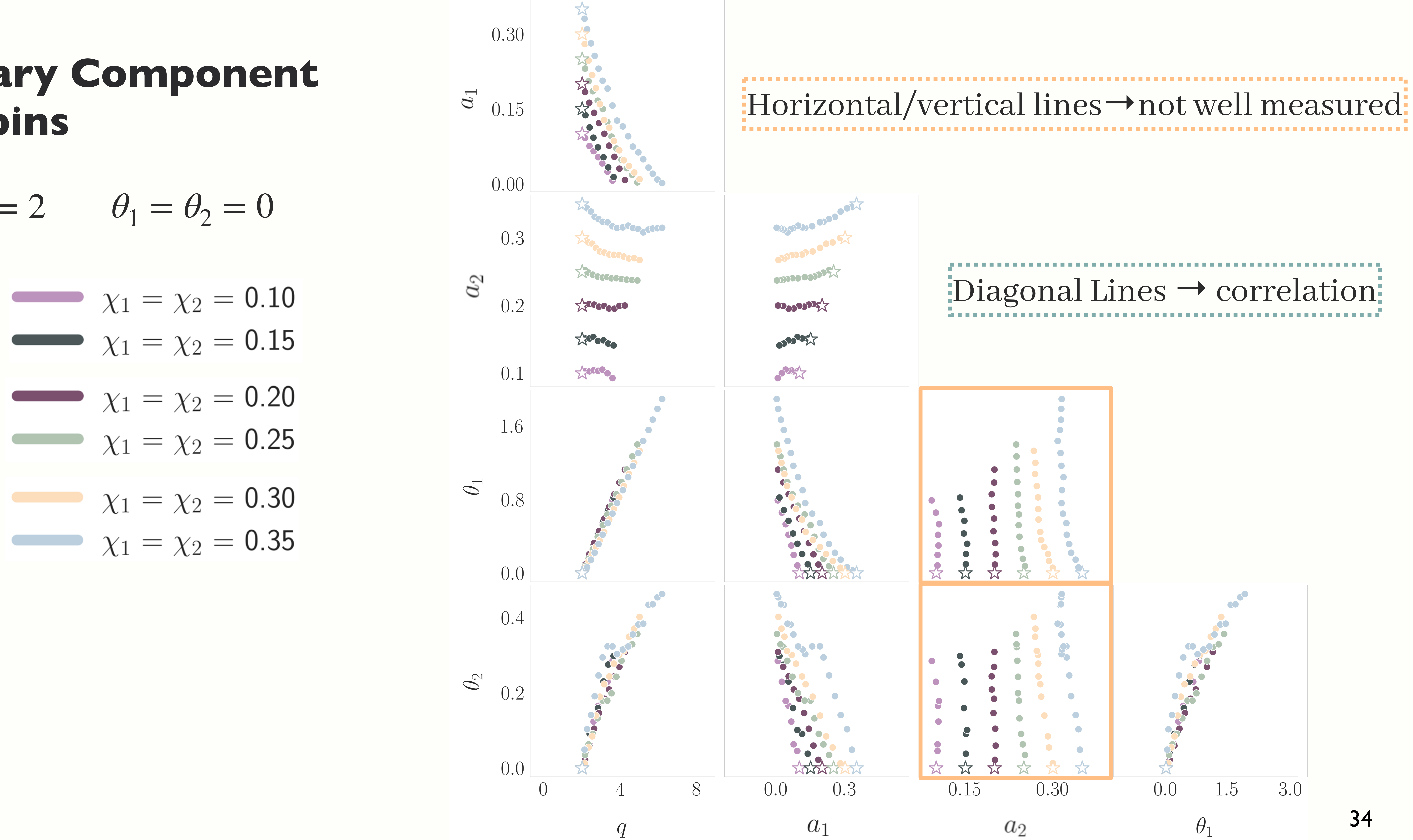
Vary Component Spins

$q = 2 \qquad \theta_1 = \theta_2 = 0$



Vary Component Spins

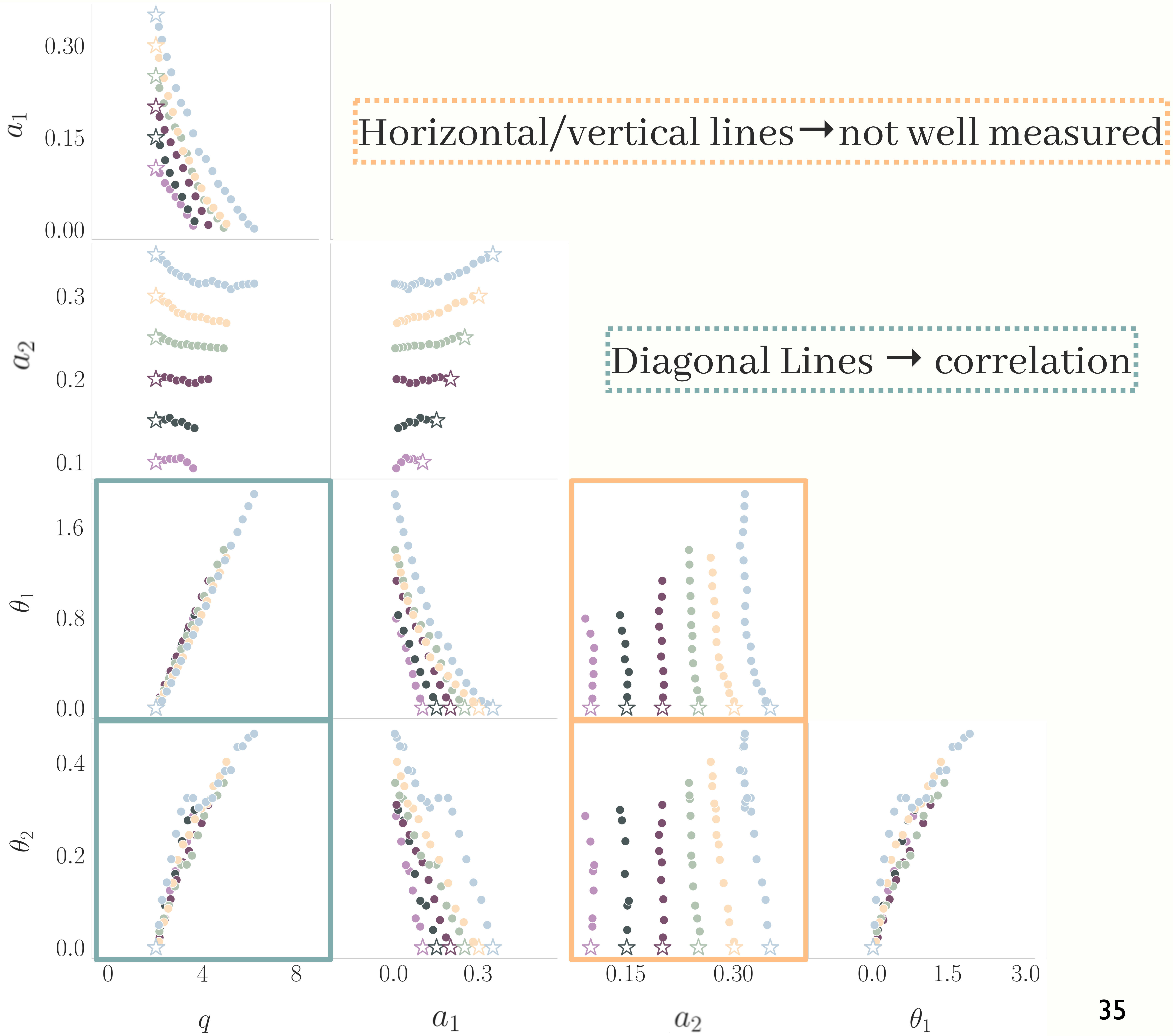
$q = 2 \qquad \theta_1 = \theta_2 = 0$



Vary Component Spins

$q = 2 \qquad \theta_1 = \theta_2 = 0$

- $\chi_1 = \chi_2 = 0.10$
- $\chi_1 = \chi_2 = 0.15$
- $\chi_1 = \chi_2 = 0.20$
- $\chi_1 = \chi_2 = 0.25$
- $\chi_1 = \chi_2 = 0.30$
- $\chi_1 = \chi_2 = 0.35$



Starting Injections ($q = 2.00$, $\theta_1 = \theta_2 = 0.00$)
$\chi_1 = \chi_2 = 0.10$
$\chi_1 = \chi_2 = 0.15$
$\chi_1 = \chi_2 = 0.20$
$\chi_1 = \chi_2 = 0.25$
$\chi_1 = \chi_2 = 0.30$
$\chi_1 = \chi_2 = 0.35$
Horizontal/vertical lines → not well measured
Diagonal Lines → correlation
a_1
a_2
θ_1
θ_2
q
$\mathcal{MM}$
$\chi_{\rm eff}$
η

Future Work

- Fix issues mapping points on boundaries
- Map correlations for high mass systems using the new network
  - Compare results
- Test mapping GW190521 maximum posterior region draws
- Analyze correlations
  - Decide whether spin parameters can be well-measured (under what circumstances)
  - Derive analytic expressions ($\chi_{\text{eff}} - \eta$ alike) for reappearing correlations

Acknowledgements

National Science Foundation
LIGO Laboratory
Caltech SFP
Katerina, Simona, and Deborah
Alan & Derek
Everyone in LIGO SURF!
:)

Questions?

Theory of Matched Filtering

Define inner product between two functions of time $a(t), b(t)$ as

$$\langle a \mid b \rangle \equiv 2 \int_0^\infty df \, \frac{\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)}{S_h(f)}$$

$$= 4 \, \mathrm{Re} \left[\int_0^\infty df \, \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_h(f)} \right].$$

Where $\tilde{a}(f)$ is the Fourier Transform of $a(t)$

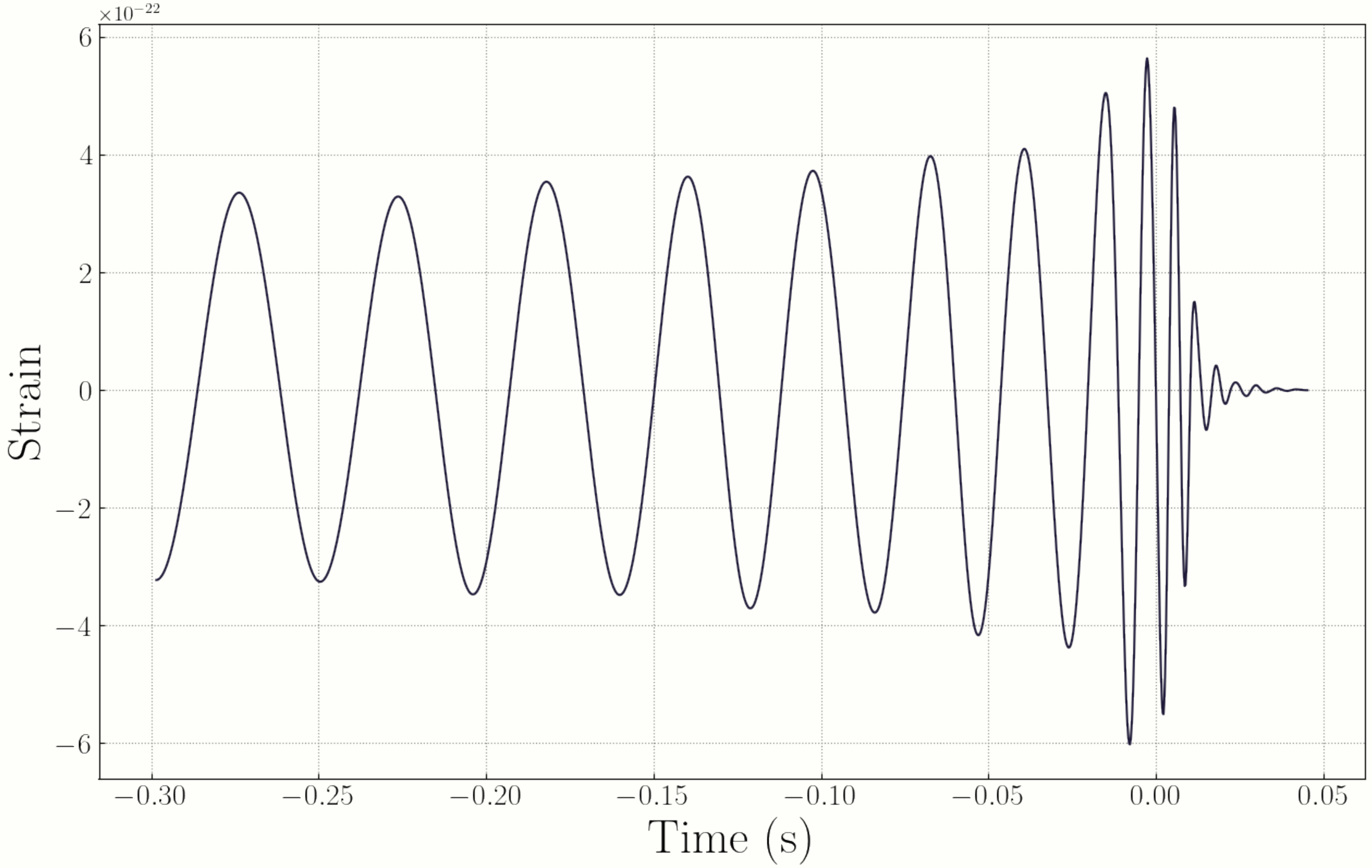
$$\tilde{a}(f) \equiv \int_{-\infty}^{\infty} dt e^{i2\pi f t} a(t)$$

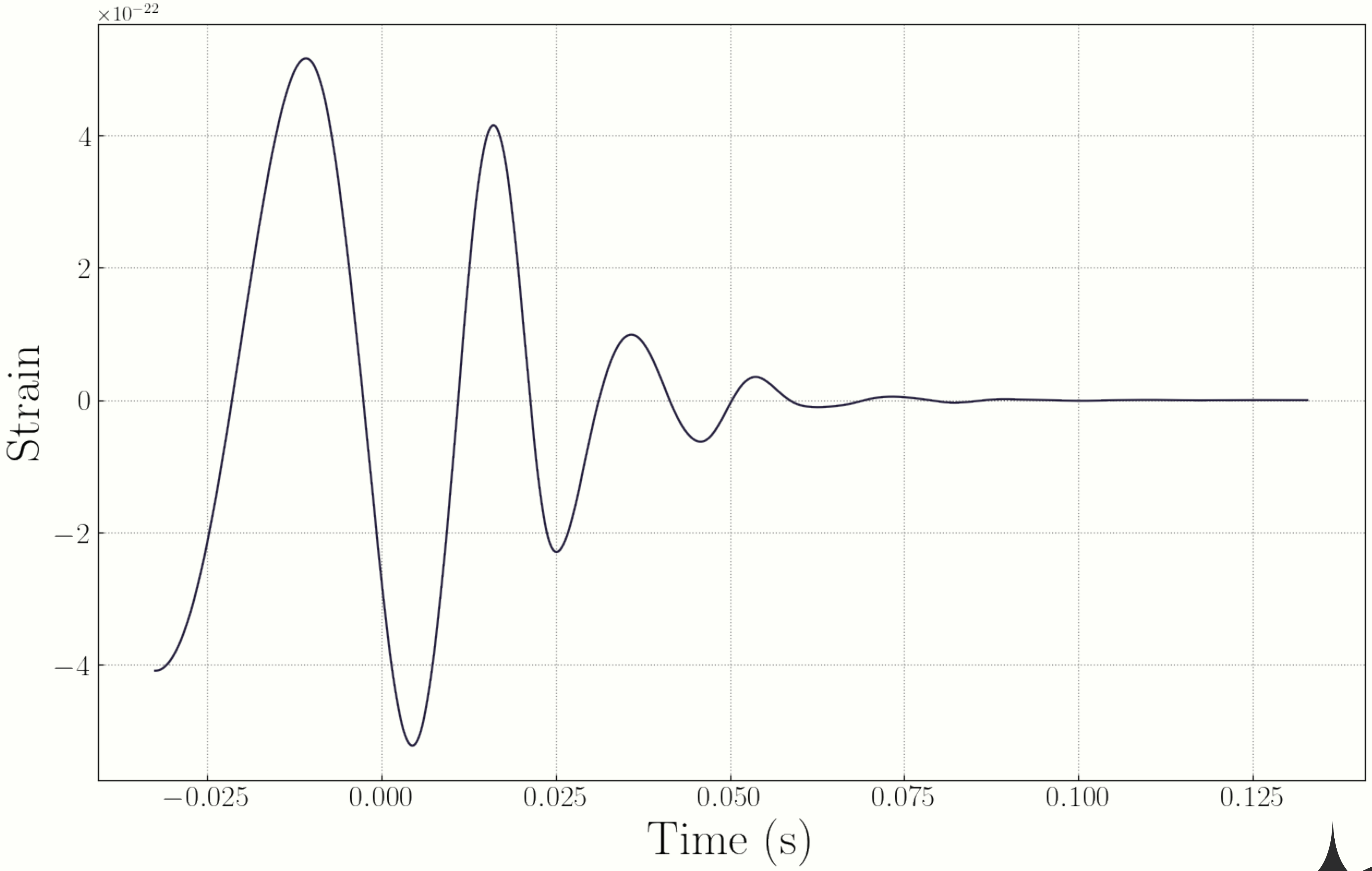
Training Data

$f_{low} = 20Hz, f_{ref} = 1840Hz$

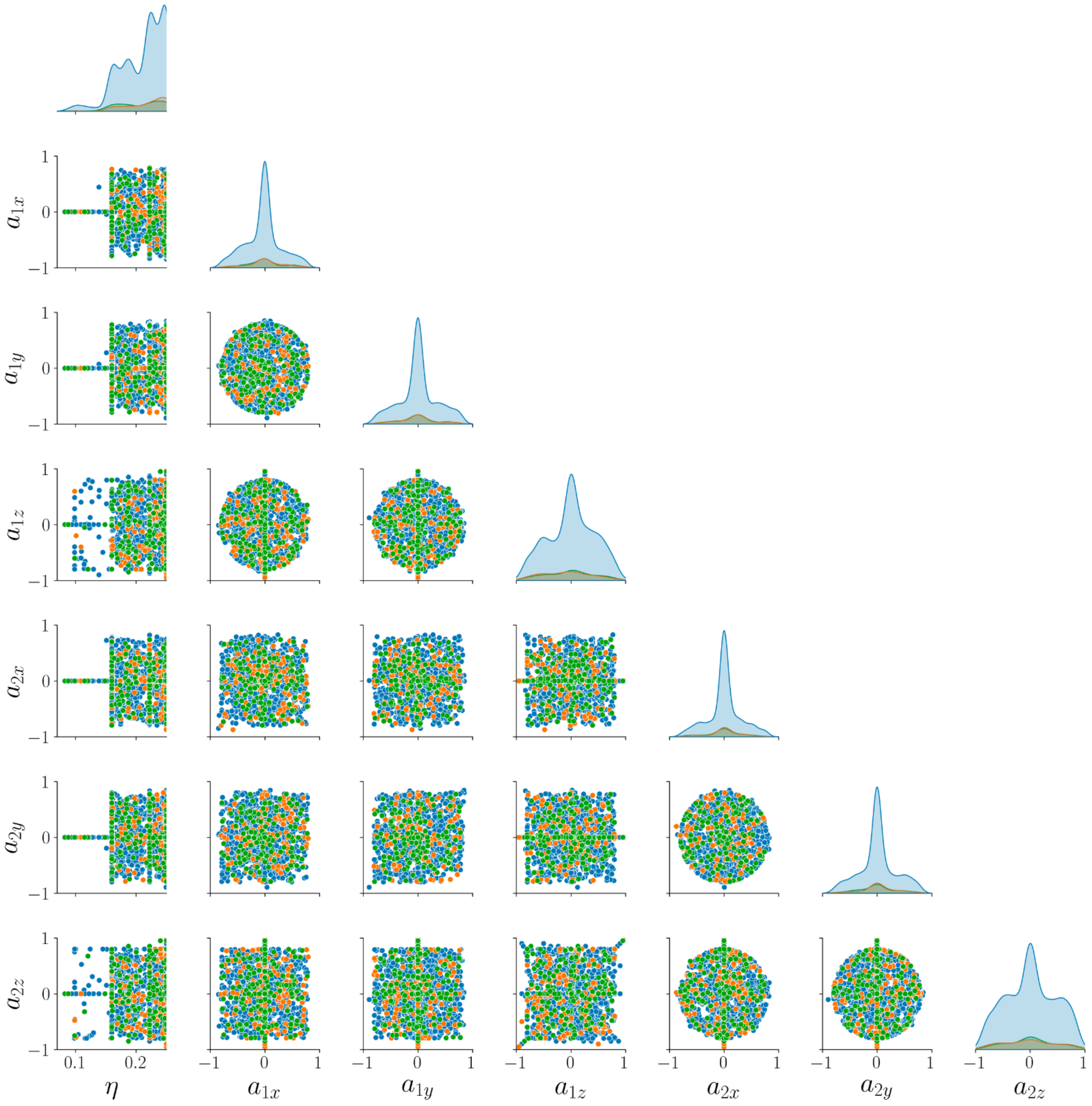
	Published	Updated
Waveforms	SXS Catalog	NRSur7dq4
Modes	$l = 2, m = 2$	All modes
Mass ratios	$0.0826 \leq \eta \leq 0.25$	$0.1224 \leq \eta \leq 0.25$
Detector Frame Mass	$92M_\odot$	$270M_\odot$

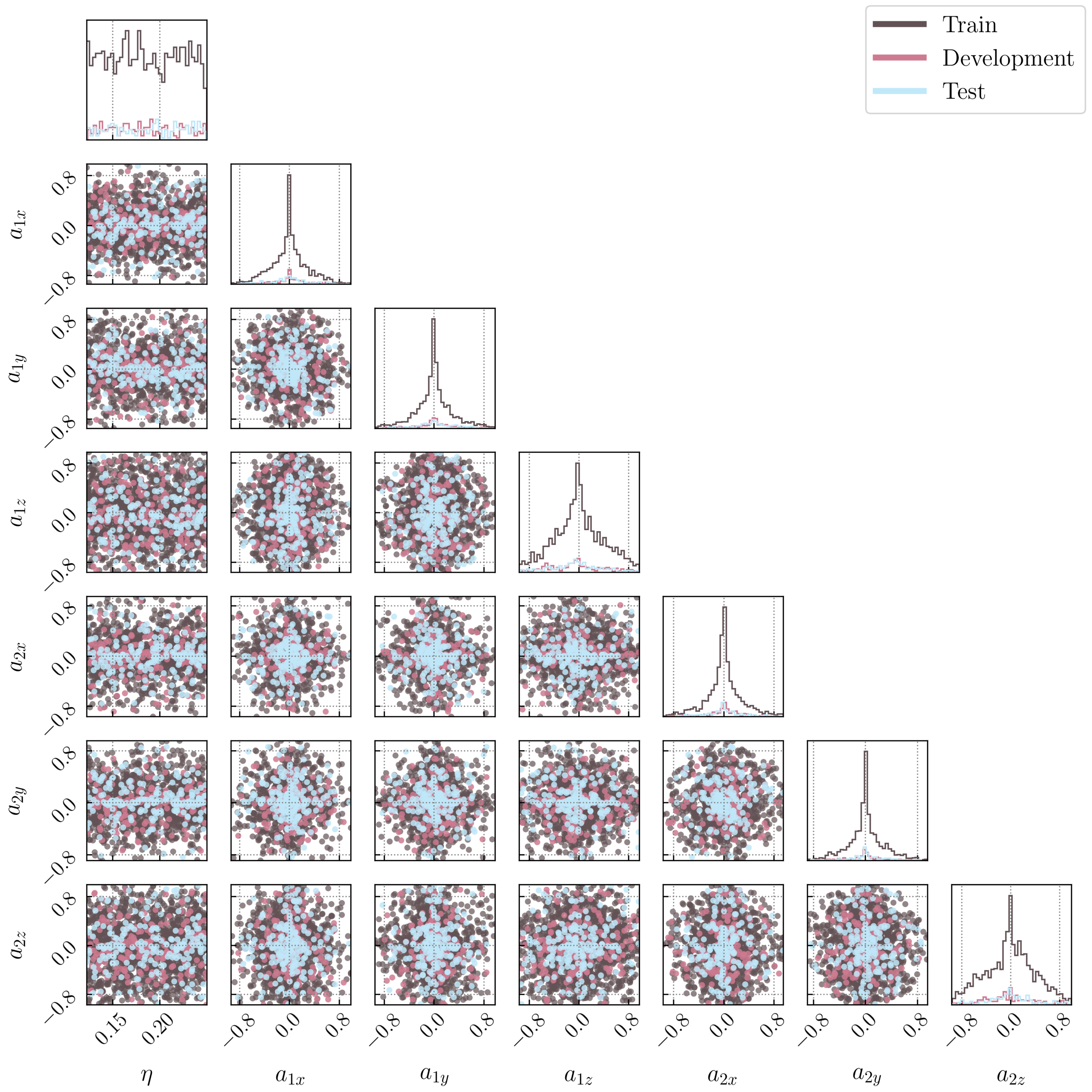
Updating Network: High Mass Systems



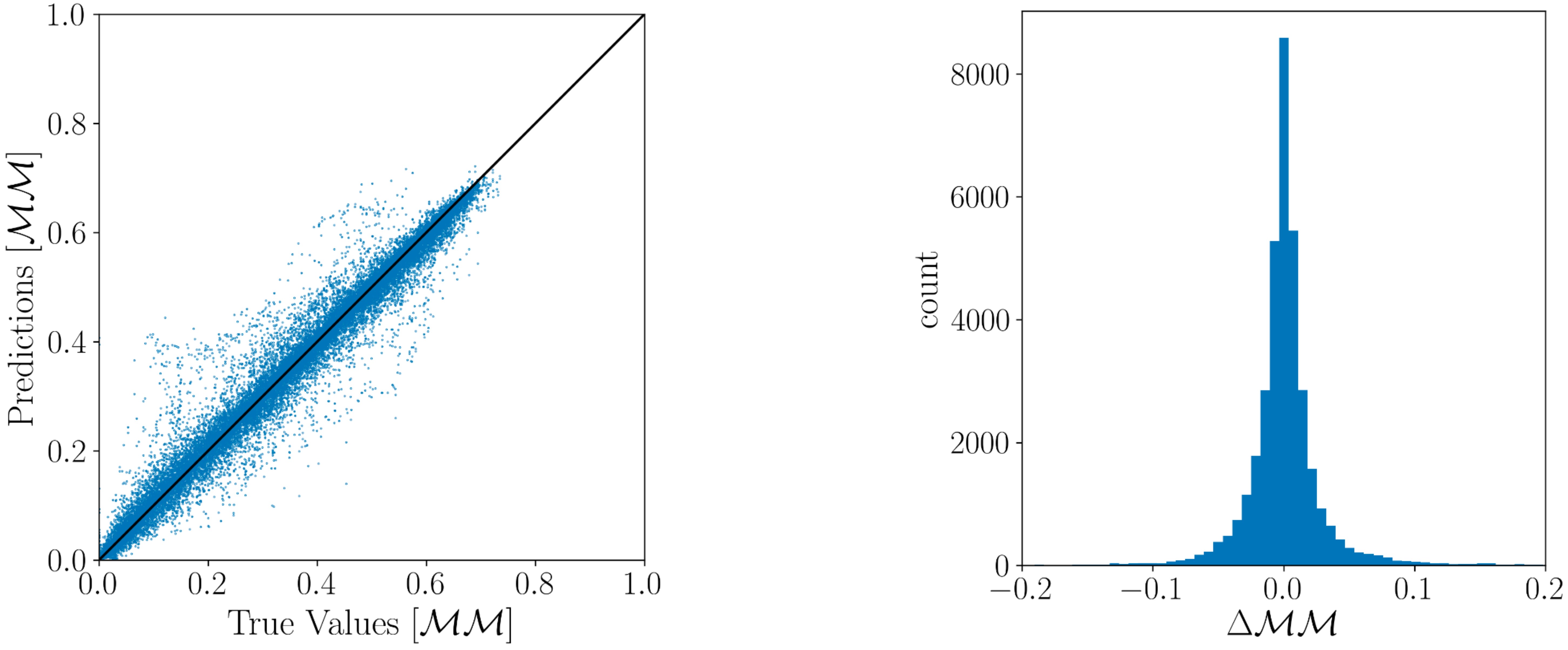


Parameter Space Coverage

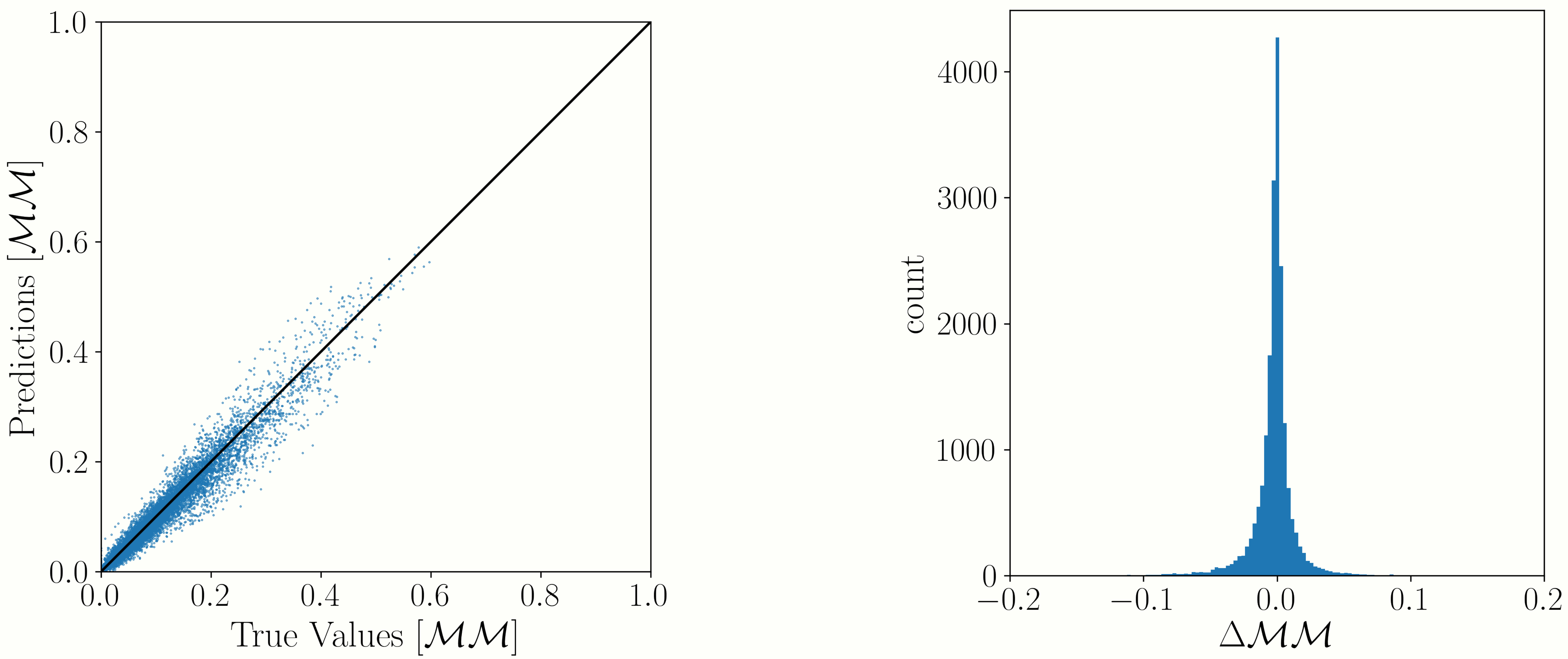




Published

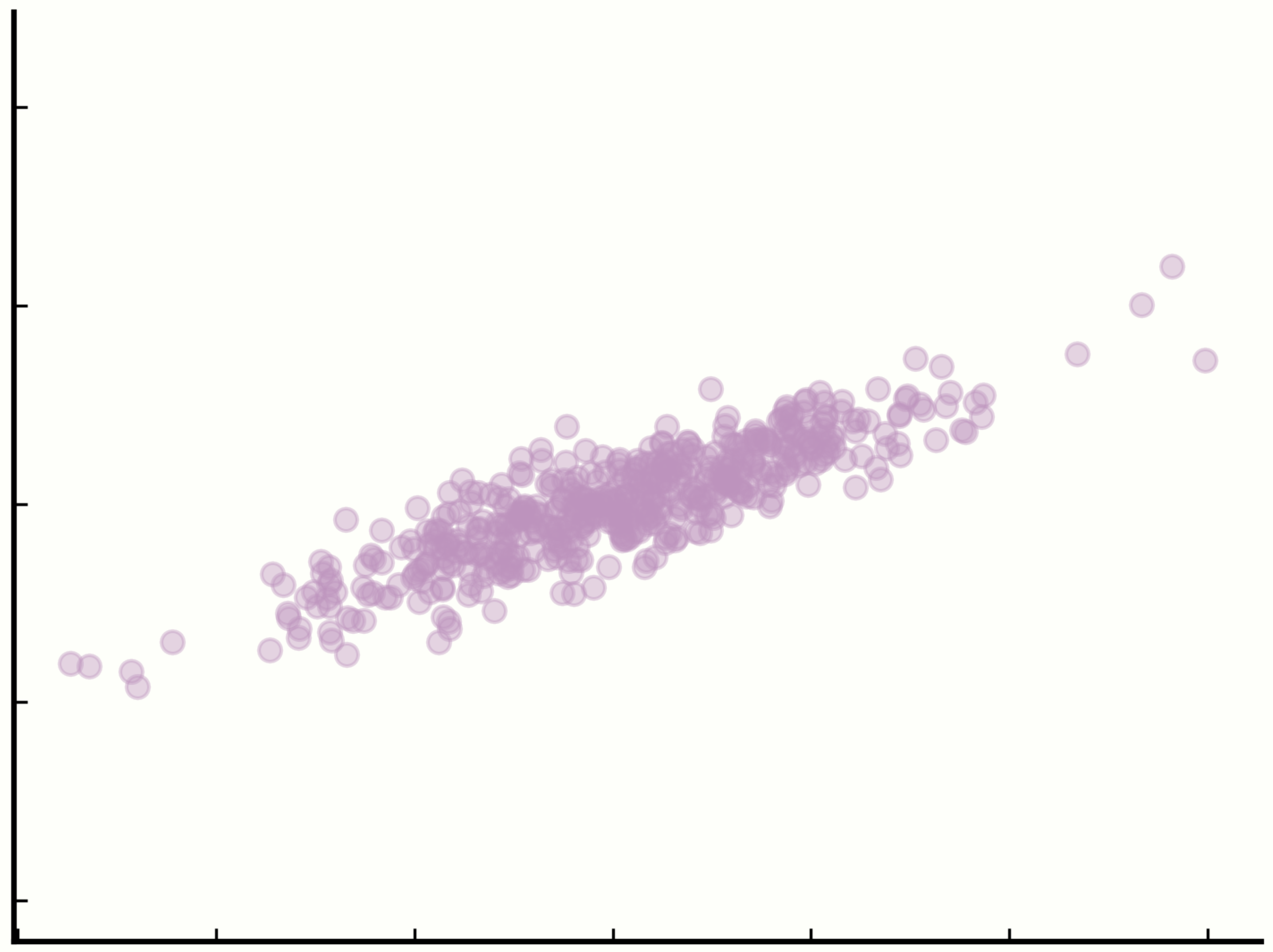


Updated

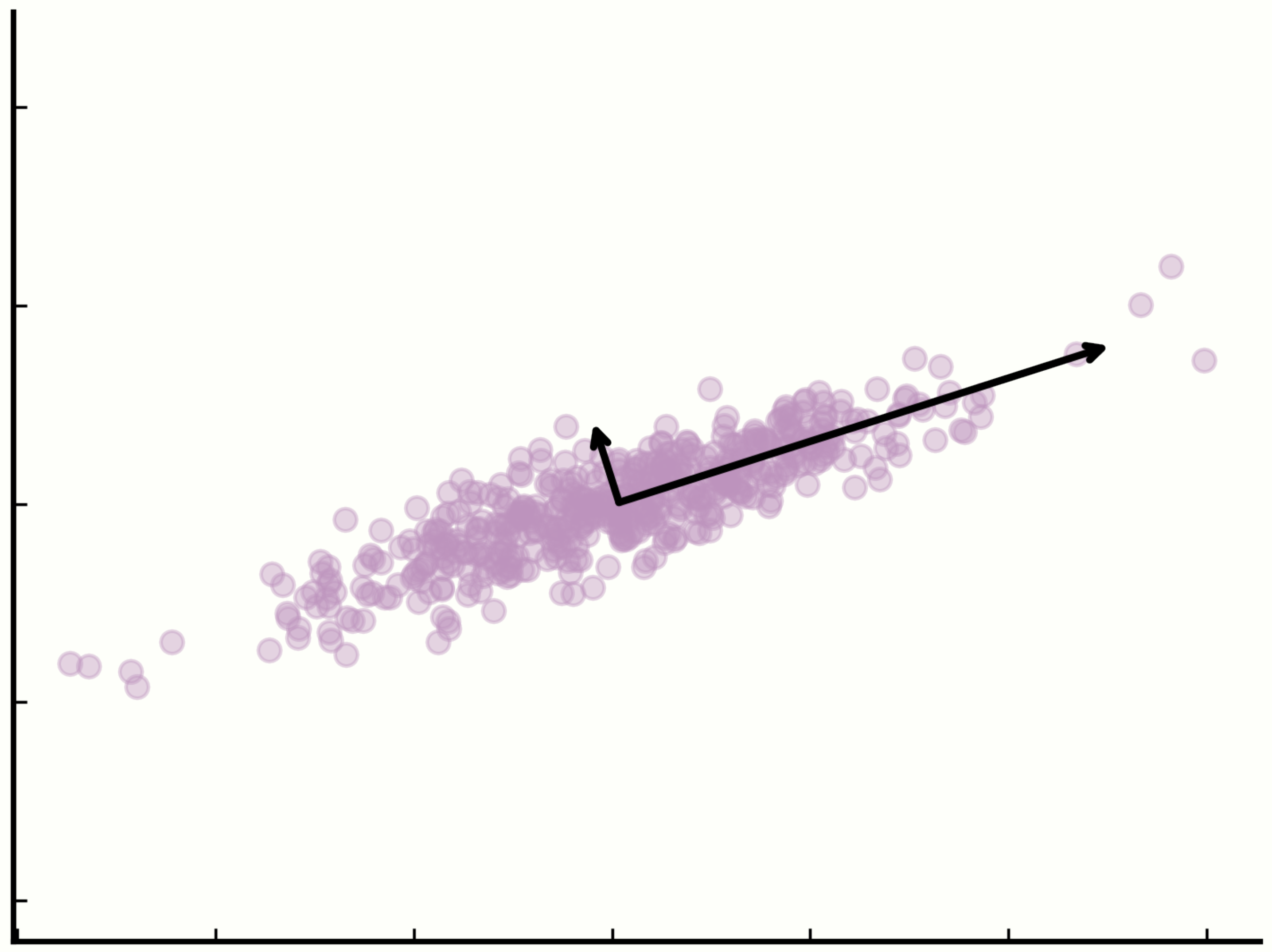


Modeling Spread of Distribution: 2D Example

Distribution



Model the spread with vectors



How do we find these vectors?

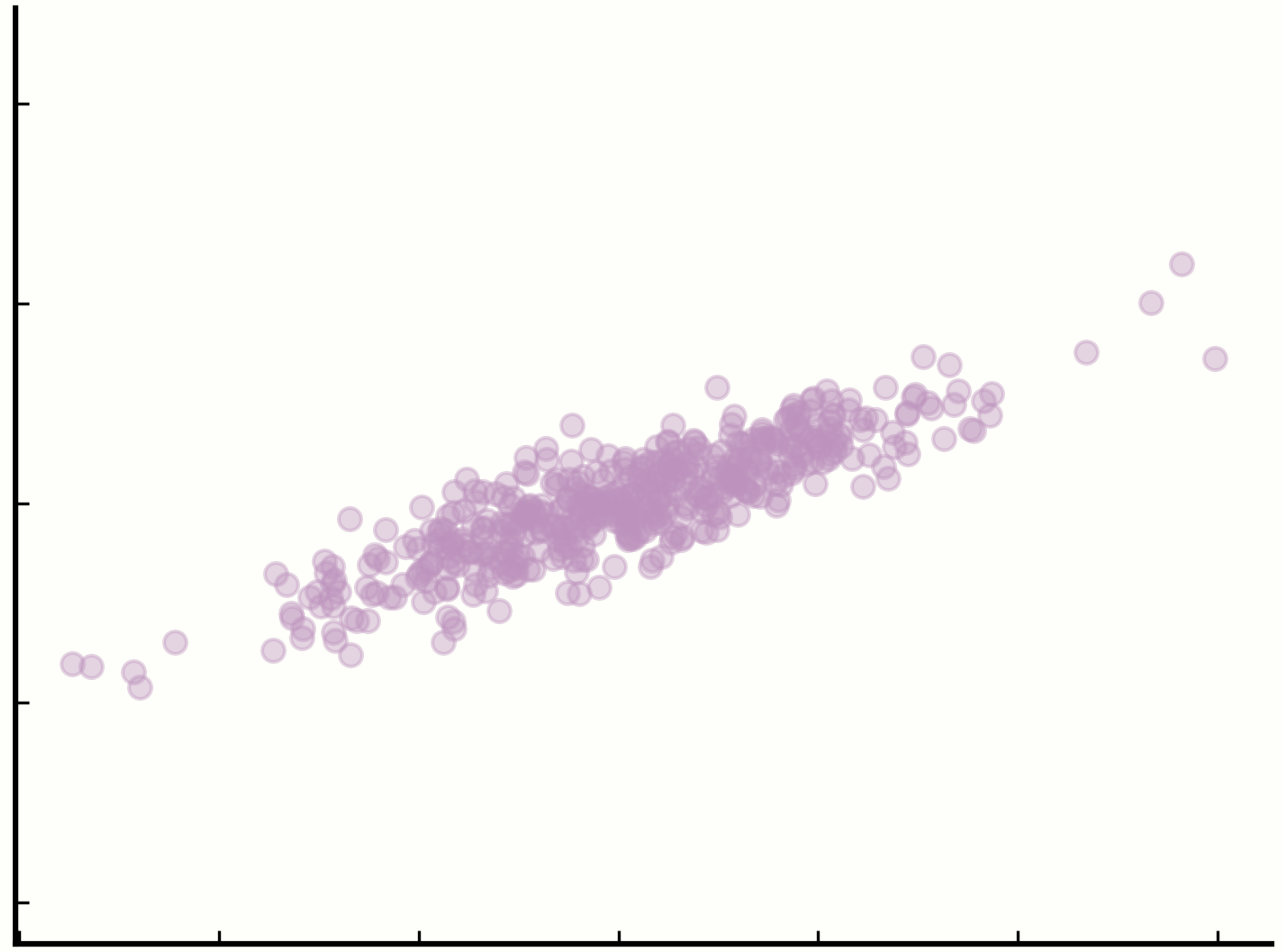
Recall: Eigenvectors in Linear Algebra

Eigenvalues λ are roots to the characteristic equation

$$\det\left(\lambda \mathbf{I}_n - \mathbf{A}\right) = 0$$

If λ is an eigenvalue of $\mathbf{A}$, then there exist non-zero $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$ such that

$$\mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$



$\mathbf{x}$ is an eigenvector of $\mathbf{A}$ with corresponding eigenvalue λ

Recall: Eigenvectors in Linear Algebra

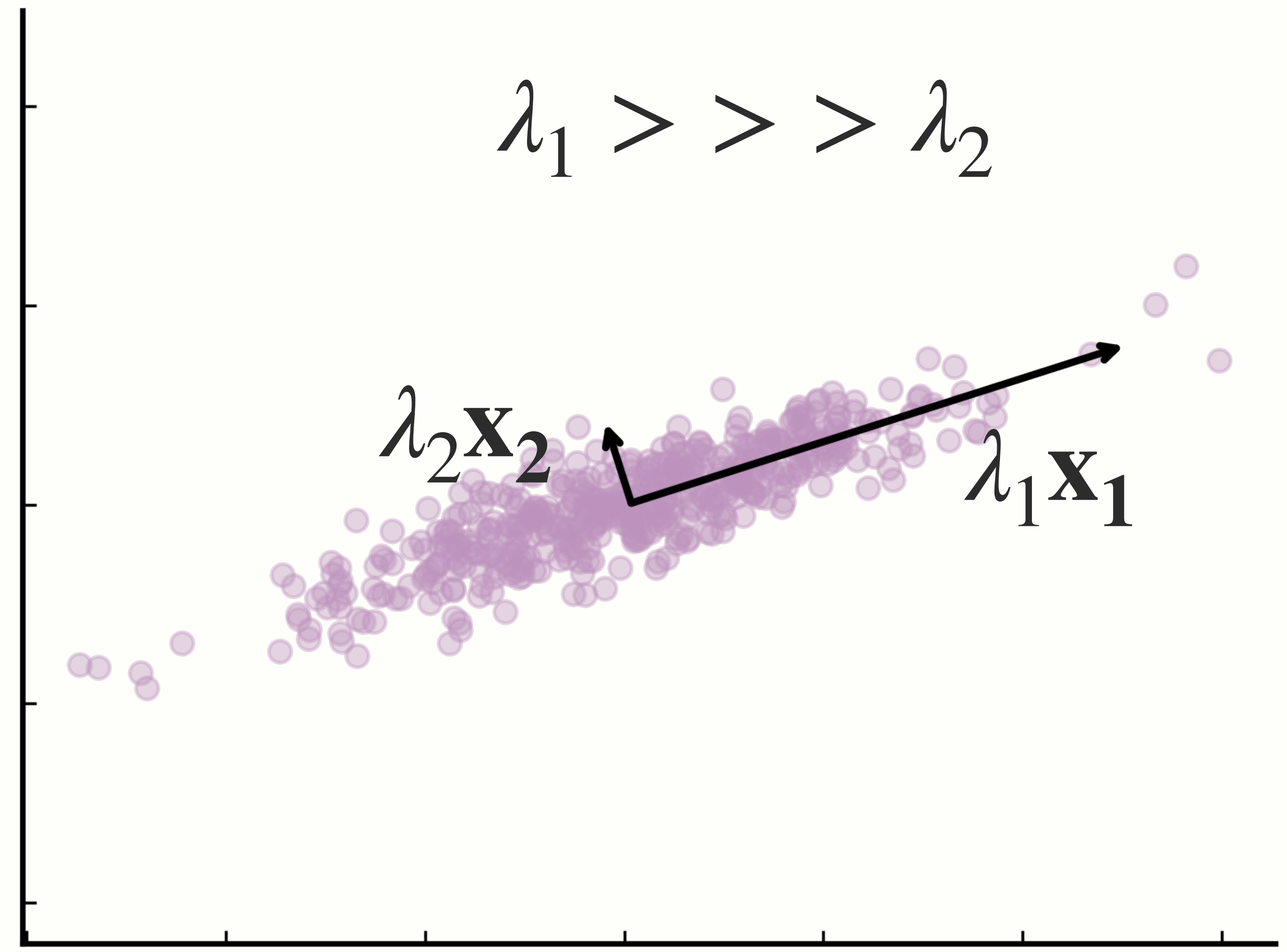
Eigenvalues λ are roots to the characteristic equation

$$\det\left(\lambda \mathbf{I}_n - \mathbf{A}\right) = 0$$

If λ is an eigenvalue of $\mathbf{A}$, then there exist non-zero $\mathbf{x} \in \mathbb{R}^{\mathbf{n}}$ such that

$$\mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

$\mathbf{x}$ is an eigenvector of $\mathbf{A}$ with corresponding eigenvalue λ



Principal Component Analysis (PCA) & Bayesian Gaussian Modeling (BGM)

BGM

Fitting Gaussian to 2D distribution

Apply Bayes' Theorem

$$p(\boldsymbol{\mu}, \boldsymbol{\Sigma} \mid \mathbf{x}) = \frac{\mathbf{p}(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) \cdot \mathbf{p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})}{\mathbf{p}(\mathbf{x})}$$

$$p(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{\mathbf{1}}{\mathbf{2}\pi |\boldsymbol{\Sigma}|^{\mathbf{1/2}}} \exp\left(-\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-\mathbf{1}}(\mathbf{x} - \boldsymbol{\mu})\right)$$

Data $\mathbf{x} = [\mathbf{x}, \mathbf{y}]$, mean $\boldsymbol{\mu} = [\mu_x, \mu_y]$, covariance $\boldsymbol{\Sigma}$

PCA

Let data $\mathbf{X}$ be an $n \times p$ matrix,

features

data points

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}$$

Principal Component Analysis (PCA) & Bayesian Gaussian Modeling (BGM)

BGM

Fitting Gaussian to 2D distribution

Apply Bayes' Theorem

$$p(\boldsymbol{\mu}, \boldsymbol{\Sigma} \mid \mathbf{x}) = \frac{\mathbf{p}(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) \cdot \mathbf{p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})}{\mathbf{p}(\mathbf{x})}$$

$$p(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{2\pi |\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$

Data $\mathbf{x} = [\mathbf{x}, \mathbf{y}]$, mean $\boldsymbol{\mu} = [\mu_x, \mu_y]$, covariance $\boldsymbol{\Sigma}$

Eigen-Decomposition(ED)

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^\mathbf{T}$$

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix} = [\mathbf{e_1} \quad \mathbf{e_2}] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \mathbf{e_1} \\ \mathbf{e_2} \end{bmatrix}$$

PCA

Let data $\mathbf{X}$ be an $n \times p$ matrix,

\# features

\# data points

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}$$

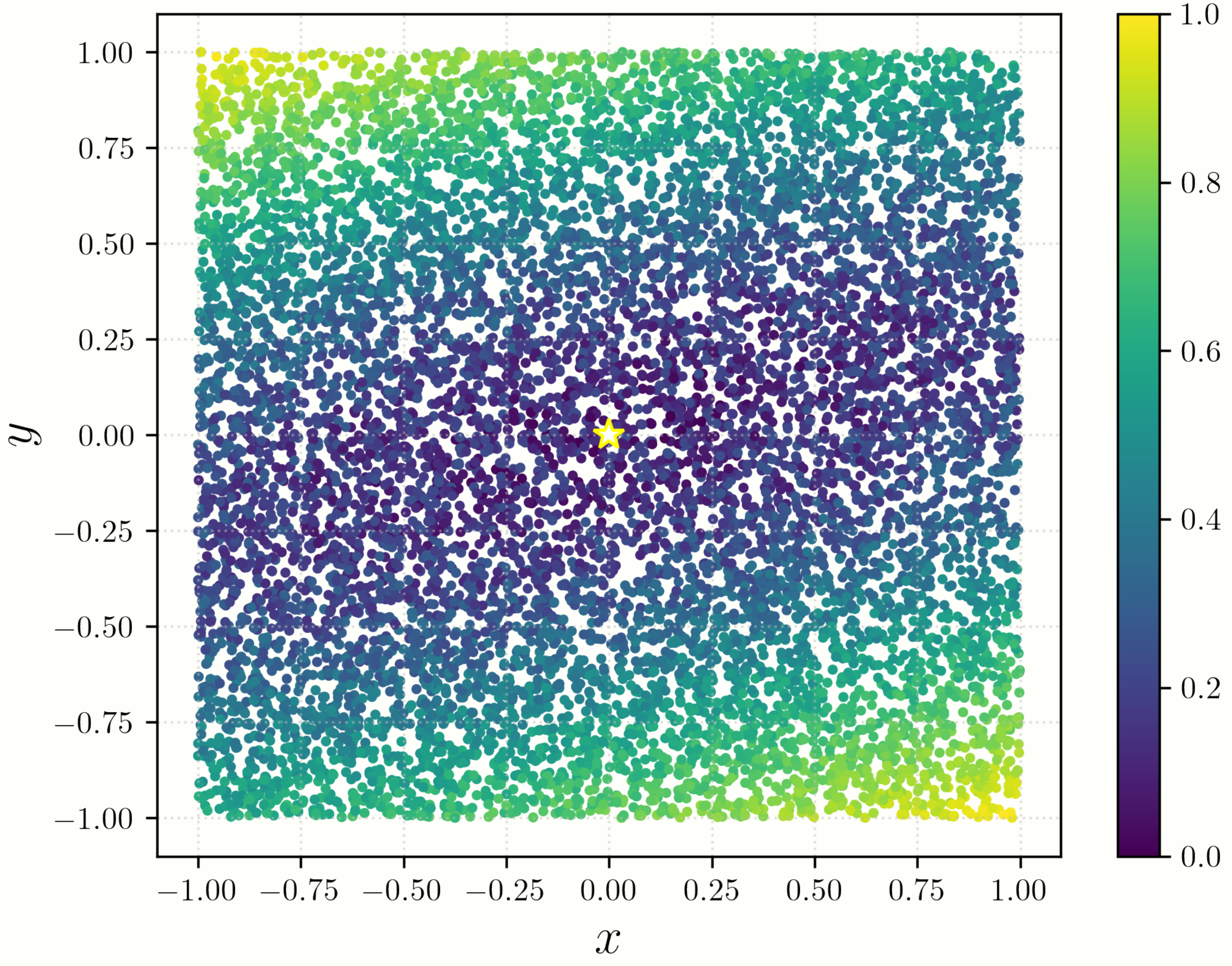
Singular Value Decomposition (SVD)

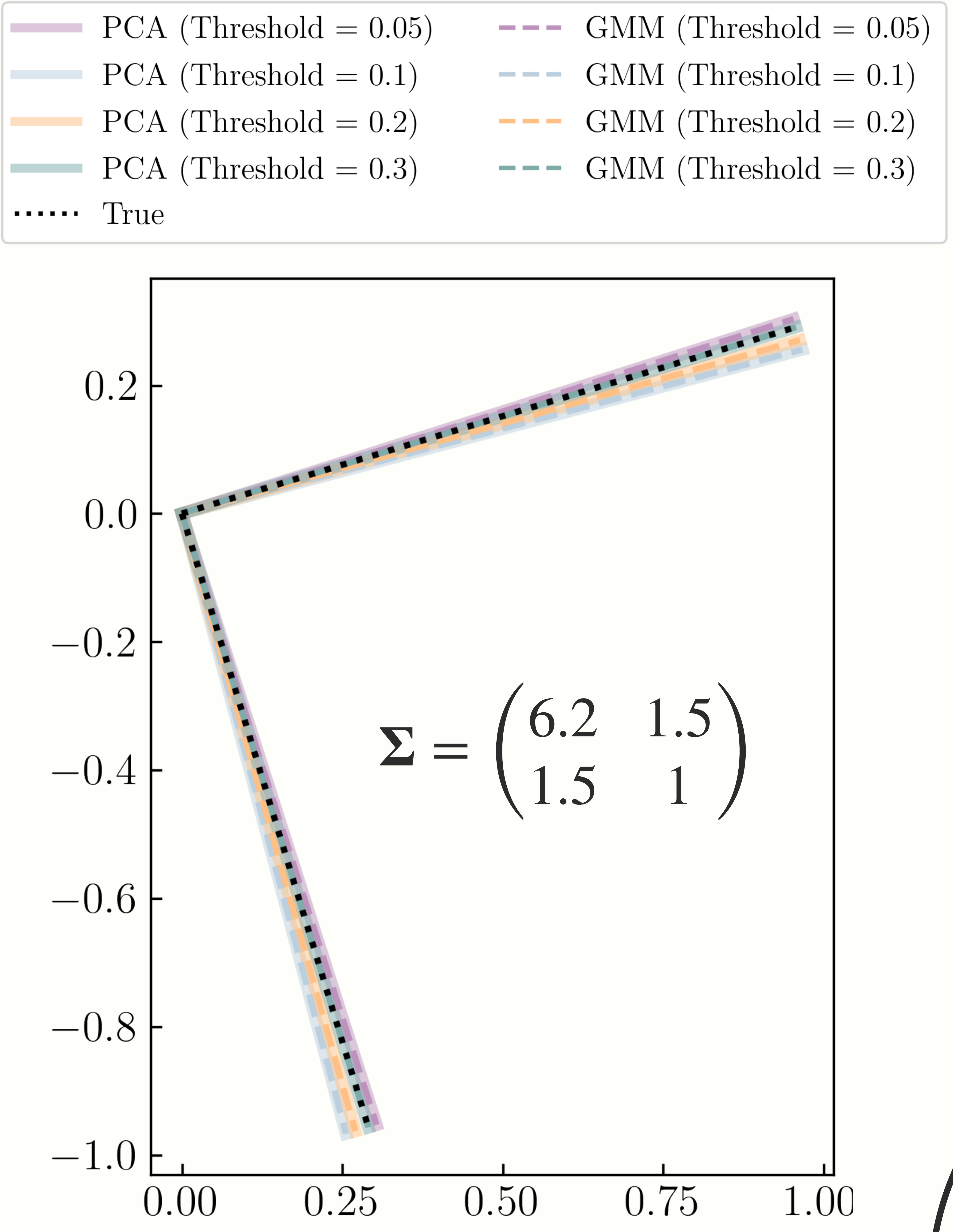
$$\mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\mathbf{T}$$

For principal components $\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_n$, $\tilde{\mathbf{x}}_i = \mathbf{X}\mathbf{v}_i$ with variance σ_i^2 $(n = 2)$

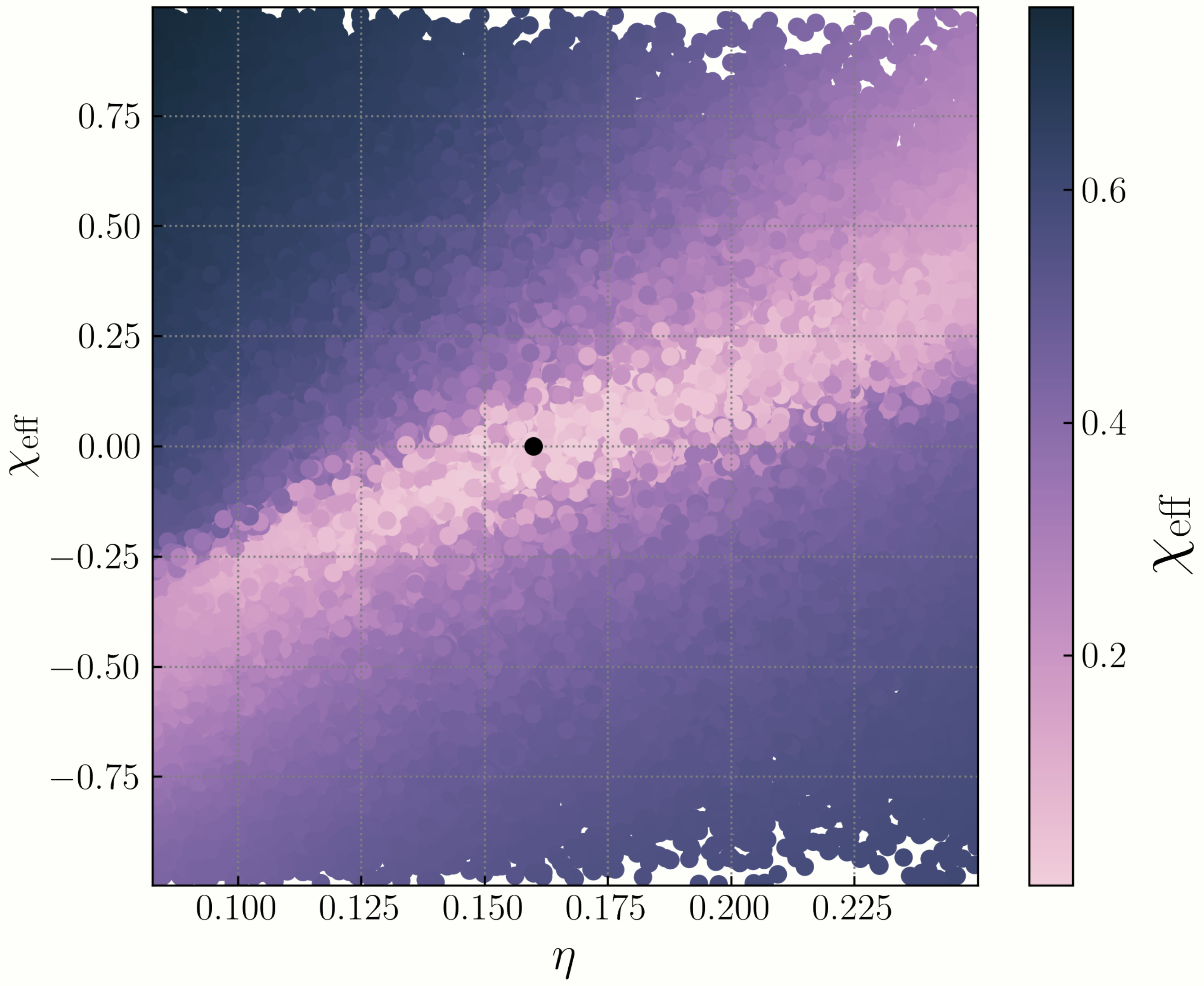
Verifying Effectiveness

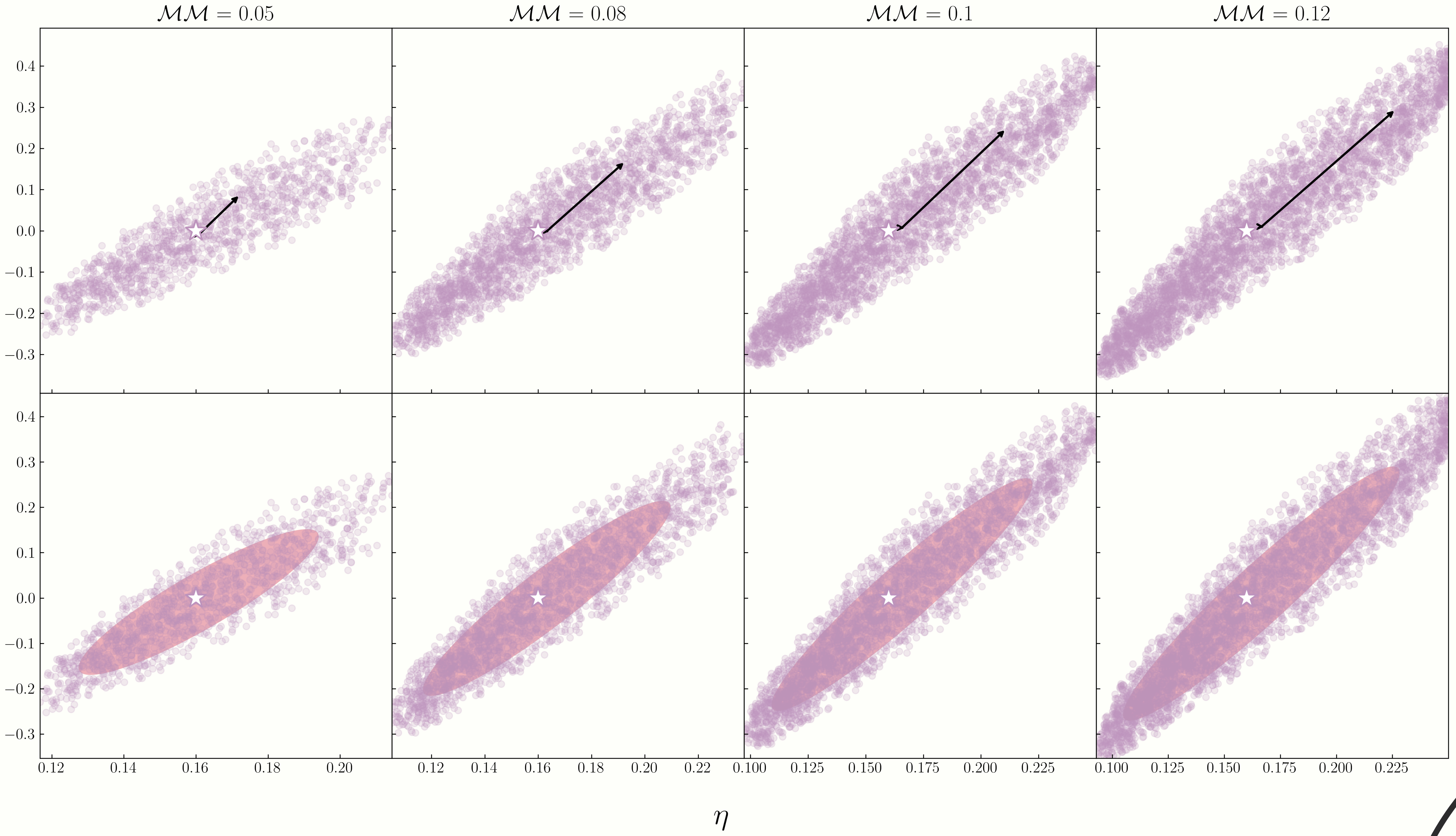
```
n_samples = 10000
Gaussian values adjusted by
noise_factor within range [-1, 1].
```



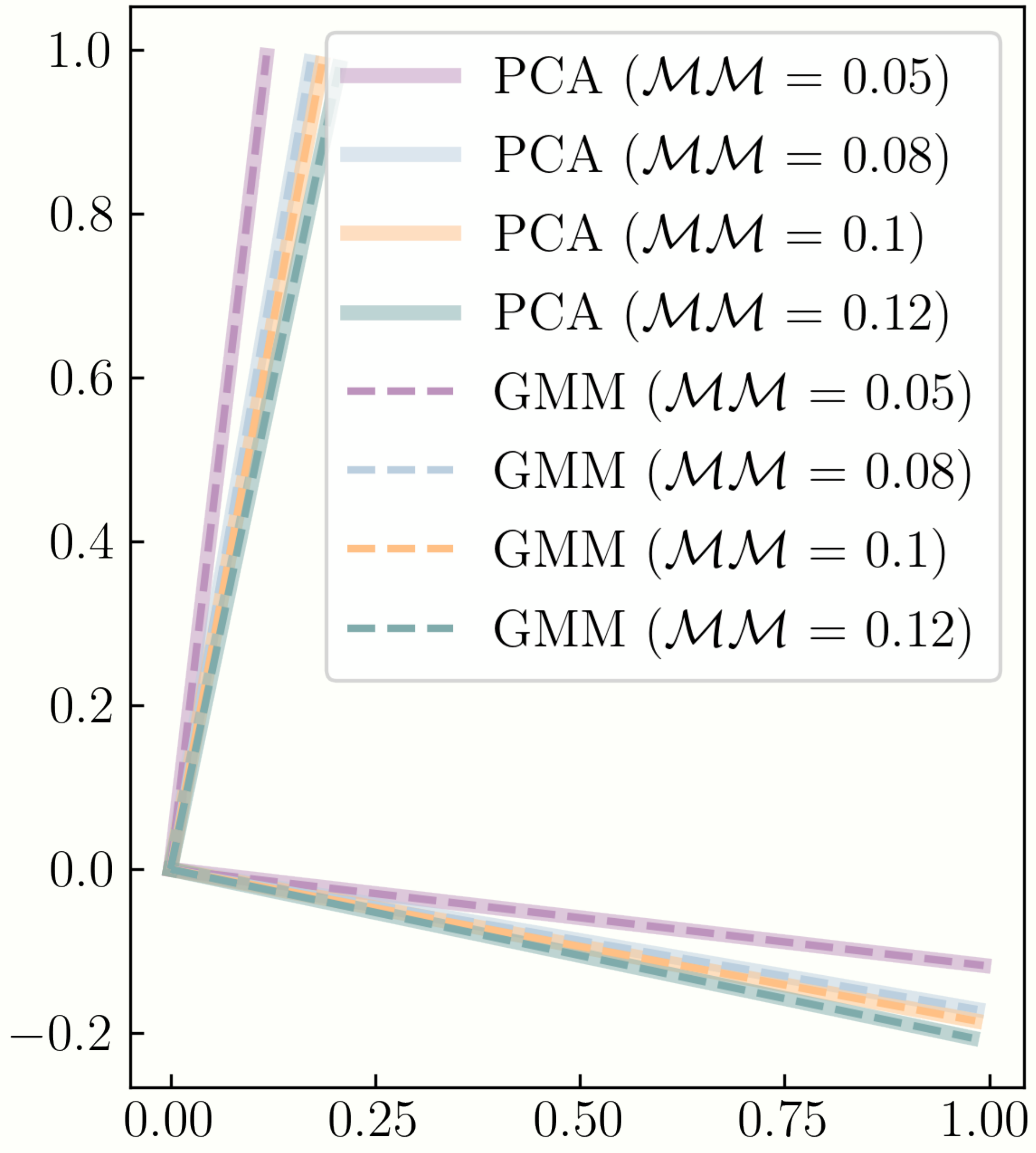


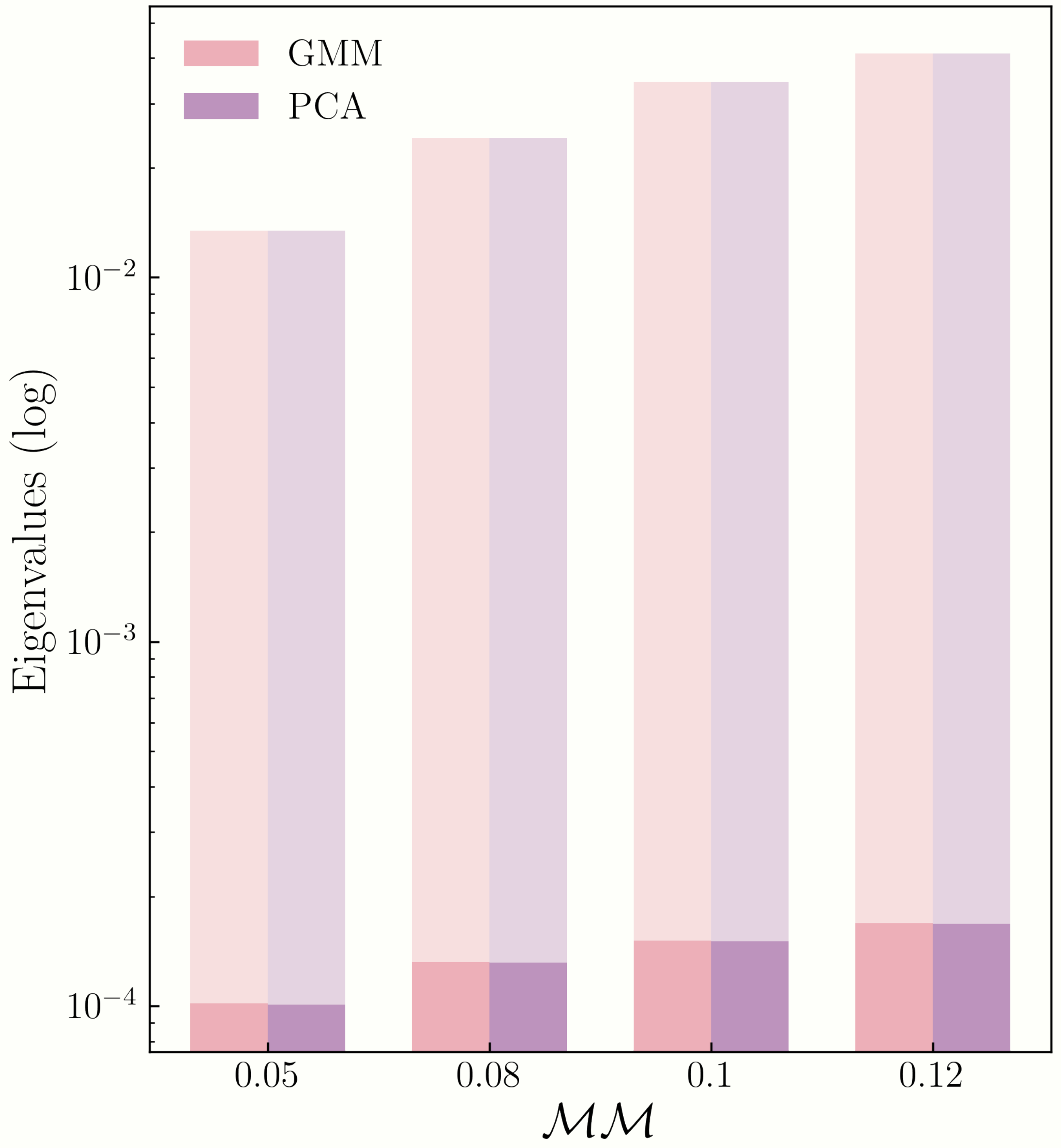
PCA-BGM Fit Comparisons





PCA-BGM Fit Comparisons Ctd.





$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{T}}$$

For principal components $\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_n,\ \tilde{\mathbf{x}}_i = \mathbf{X}\mathbf{v}_i$ with variance $\sigma_i^2\ \ (n = 2)$

$$\mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathbf{T}}$$

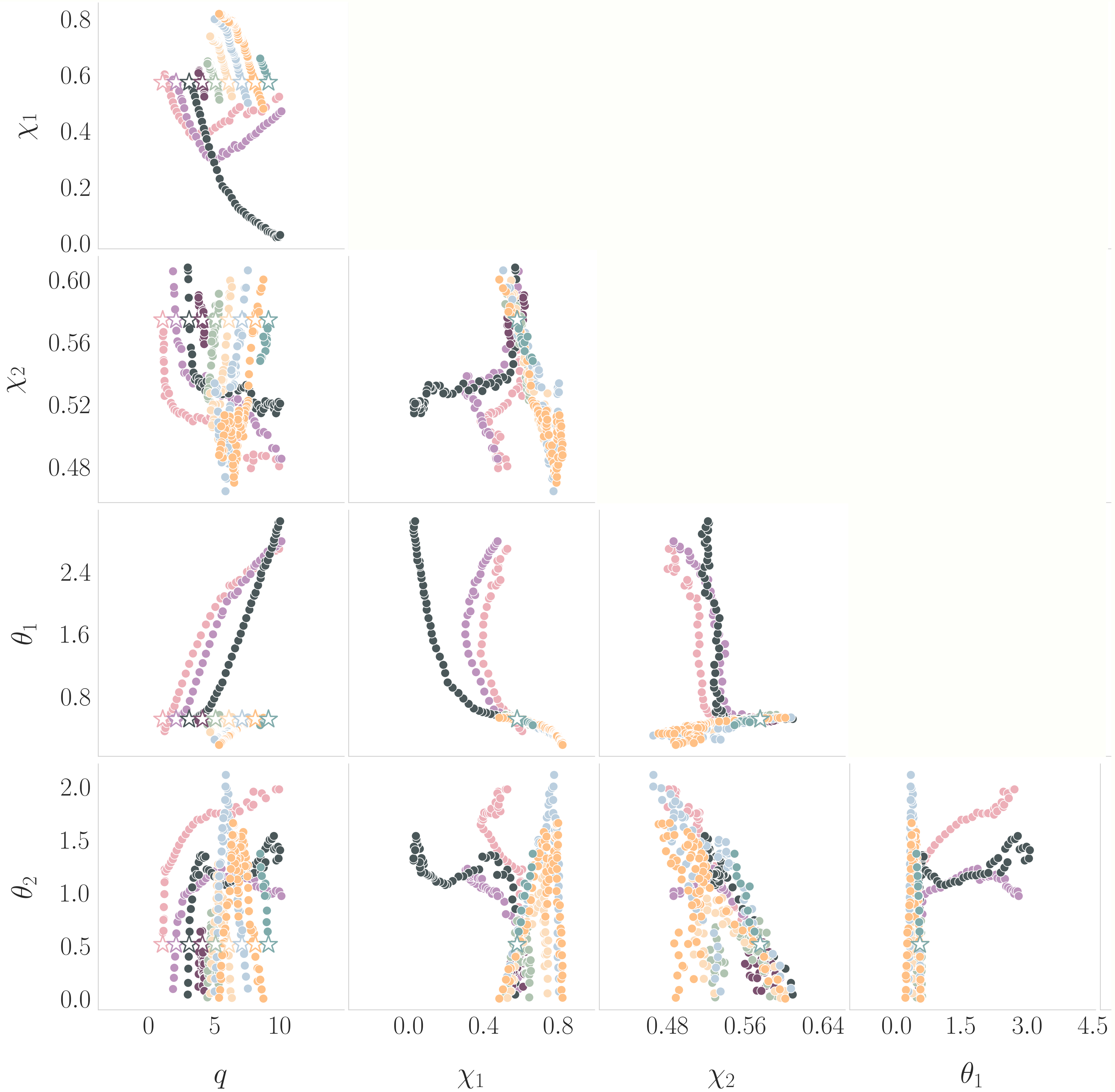
$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix} = [\mathbf{e_1} \quad \mathbf{e_2}] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \mathbf{e_1} \\ \mathbf{e_2} \end{bmatrix}$$

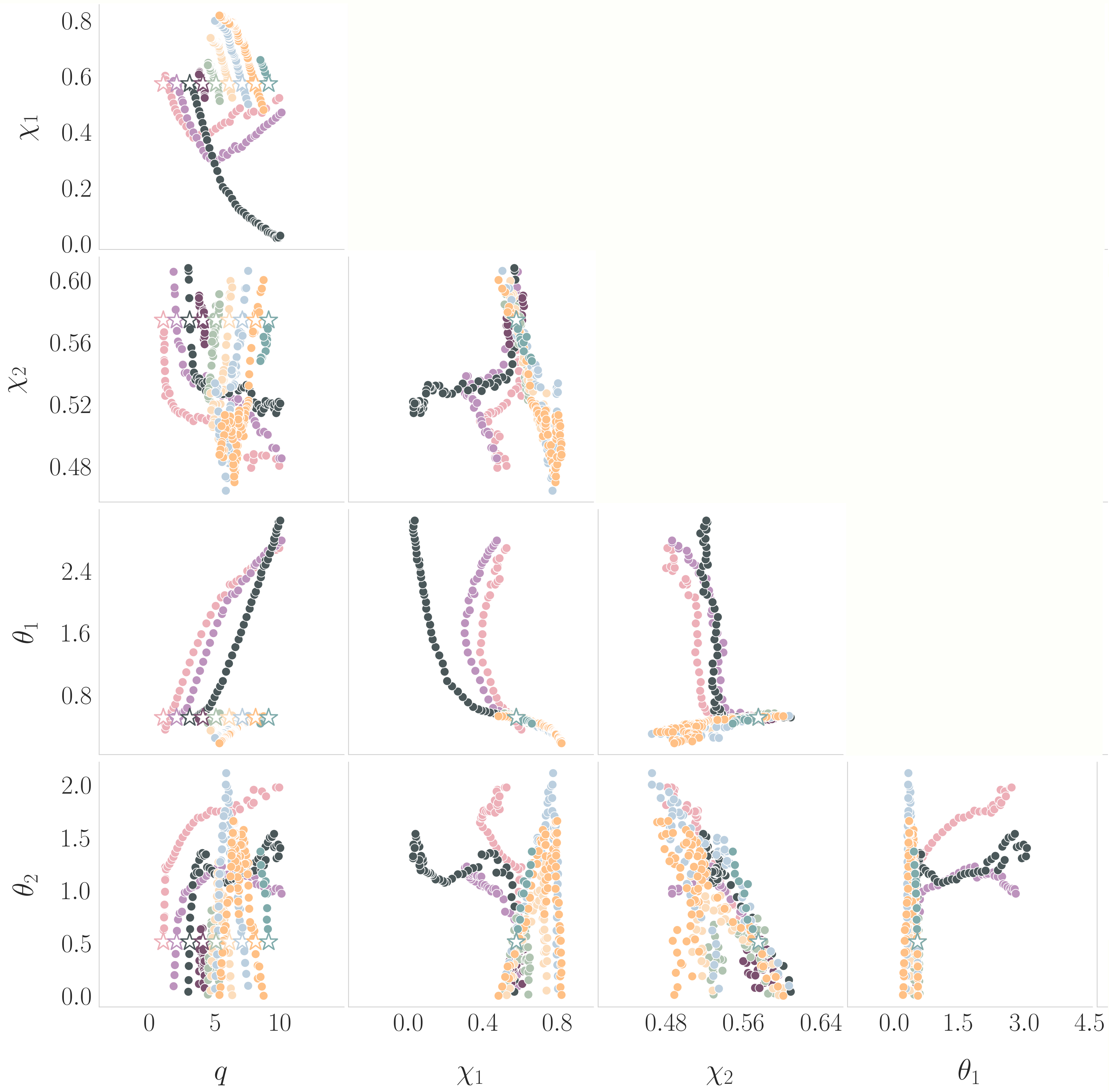
$$\sigma_i^2 \sim \lambda_i \qquad \tilde{\mathbf{x}}_\mathbf{i} \sim \mathbf{e_i}$$

Vary Mass Ratios

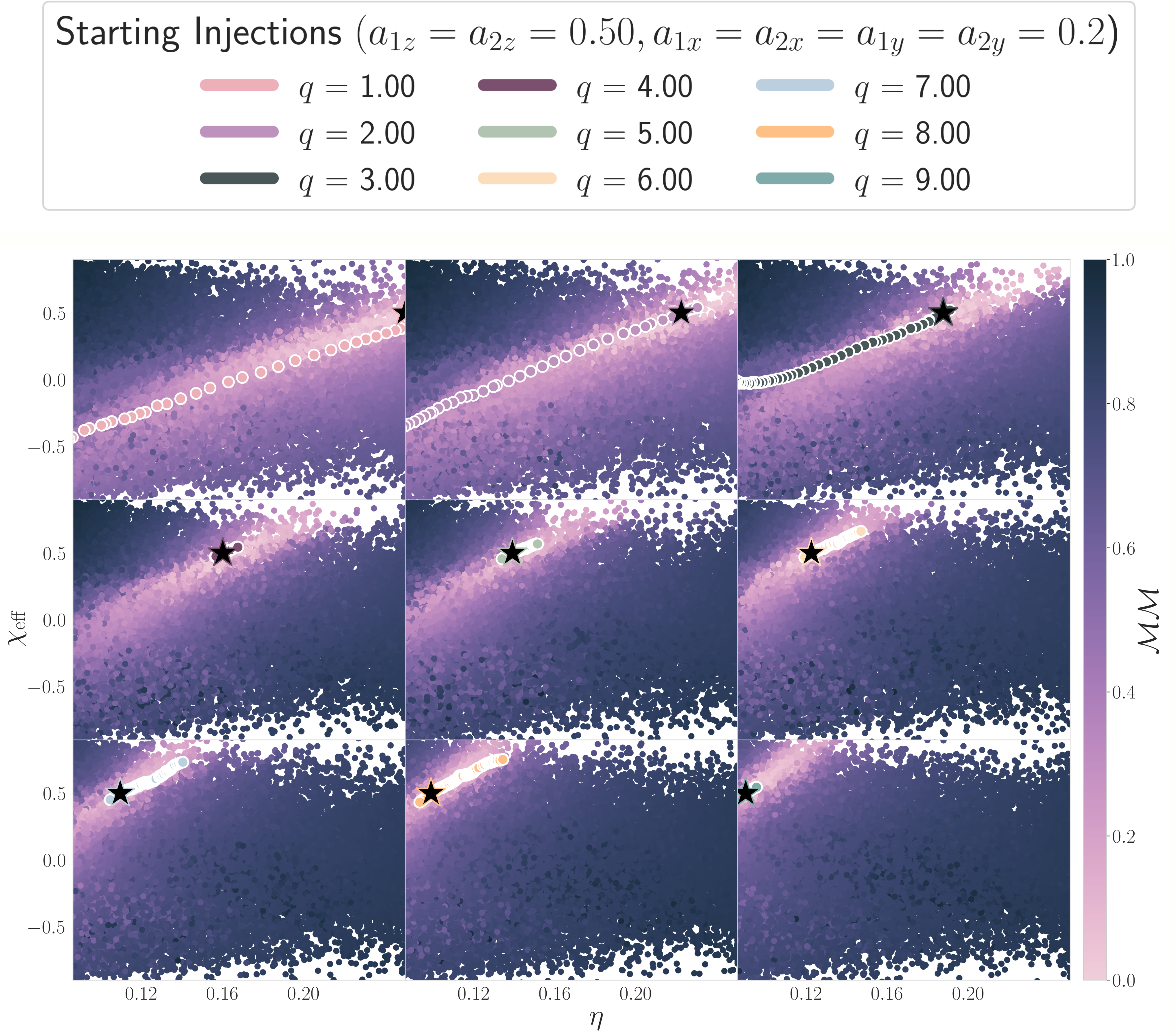
$$a_{1z} = a_{2z} = 0.50$$

$$a_{1x} = a_{2x} = a_{1y} = a_{2y} = 0.2$$





χ1
χ2
θ1
θ2
q
χ1
χ2
θ1



Starting Injections $(a_{1z} = a_{2z} = 0.50, a_{1x} = a_{2x} = a_{1y} = a_{2y} = 0.2)$
$q = 1.00$
$q = 2.00$
$q = 3.00$
$q = 4.00$
$q = 5.00$
$q = 6.00$
$q = 7.00$
$q = 8.00$
$q = 9.00$
$\chi_{\rm eff}$
η
$\mathcal{MM}$