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Identifying Correlations in Precessing Gravitational-Wave Signals with Machine Learning

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ABSTRACT

Binary binary hole (BBH) spins provides important insights on the formation environments, evolutionary history, and dynamics of these objects, which could be of interest of the broader astrophysics community (Mandel & Farmer 2022). We would like to better measure signals for highly massive (total mass $> 100M_{\odot}$, primary mass$\lesssim 500M_{\odot}$), highly spinning BBH systems, which are subject to spurious measurements due to their very short duration and low bandwidth (Abbott et al. 2020). The astrophysical parameters of gravitational wave (GW) sources are extracted from match filtering observed signals to templated waveforms. The waveforms which include the most underlying physics are those generated with numerical relativity (NR). However, different parameters of NR simulation, such as mass and spin, can lead to extremely similar waveforms. In such cases, the analysis pipeline will not be able to distinguish potential sources. We are interested in constructing a neural network to study the correlations between different parameters of waveforms with spin precession and to identify potential ways to break such degeneracies. The results produced by this network would inform us the measurability of spin parameters from inferred waveform signals.

1. INTRODUCTION

GW190521 ($M \sim 150\odot$) is the heaviest BH binary detected to date and one of the few BBHs measured to be highly precessing. It is the first strong observational evidence of intermediate-mass BH (IMBH), which is believed to be the missing link for explaining the formation of supermassive BHs (Abbott et al. 2020). The detected waveform is dominated by the merger phase where effects of precession remain elusive. In the upcoming LIGO's fourth observing run (O4) as well as in the advent of the space-based Laser-Interferometer Space Antenna (LISA) and the ground-based Einstein Telescope (ET), we expect to observe large sample of events similar to GW190521 as a result of increased detector sensitivity. Spin configurations are indicative of the compact progenitor's orbital dynamics and therefore help illuminate the formation channels of BH mergers in the pair-instability (PI) mass gap and aid population modeling of BBHs (Abbott et al. 2020; Mandel & Farmer 2022).

We would like to respond to signals from such systems with maximal accuracy, which requires a thorough phenomenological understanding of the measurability of spin parameters from inferred waveforms.

2. OBJECTIVES

The objective of the project is to identify degeneracies in the parameter space for highly massive, precessing BBH systems using machine learning. I will 1) determine whether a certain set of parameters can be recovered from detected waveform, 2) investigate the correlations between degenerate parameters, and 3) quantify and produce visualizations of such correlations. Understanding the degeneracies and correlations between spin measurements will inform us as to which spin paramameters are actually independently measurable in GW data.

3. BACKGROUND & APPROACH

3.1. *Theoretical Modeling*

My project will be largely based on the neural network `mismatch_prediction` presented in (Ferguson 2023). The model is currently trained on the SXS GW catalog. The network predicts the mismatch of the GW emitted by two BBH systems with initial input parameters $\lambda = \eta, \mathbf{a_1}, \mathbf{a_2}$, consisting of the symmetric mass ratio η

$$\eta = m_1 m_2 / (m_1 + m_2)^2 \quad 0 \leq \eta \leq 0.25 \tag{1}$$

and the dimensionless spin vectors $\mathbf{a_1}, \mathbf{a_2}$, which are 3-dimensional vectors with x, y, z components.

$$\mathbf{a} = \frac{\mathbf{J}}{m^2} \quad 0 < a < 1$$

The model defines a mismatch metric $\mathcal{MM}$ to assess how different a resulting waveform is from an existing waveform.

$$\mathcal{MM} = 1 - \max_{t,\phi} \mathcal{O}\left[h_1, h_2\right] \equiv 1 - \max_{t,\phi} \frac{\langle h_1 \mid h_2 \rangle}{\sqrt{\langle h_1 \mid h_1 \rangle \langle h_2 \mid h_2 \rangle}}$$

where

$$\langle h_1 \mid h_2 \rangle = 2 \int_{f_0}^{\infty} \frac{h_1^* h_2 + h_1 h_2^*}{S_n} df$$

h_1, h_2 are the frequency domain strain of the waveforms, and S_n is the one-sided power spectral density of the detector. $\mathcal{MM}$ is normalized to 1 with $\mathcal{MM} = 0$ corresponding to two identical waveforms. The mismatch is computed on a flat noise curve at f_{ref}, where the spin vectors are defined. The existing network is able to identify degenerate regions in the parameter space with $l = 2, m = 2$ modes for systems of $M_{tot} = 1 M_{\odot}$, corresponding to $f_{\mathrm{ref}} = 1840$ Hz. Since the frequency of the emitted gravitational wave scales inversely with the total mass of the system, heavy binaries merge at lower frequencies, which leaves fewer cycles in the sensitive band of ground-based detectors. I will be adjusting the frequency cutoff to adapt the network specifically to highly massive, precessing BBHs with fewer observable cycles. To account for effects of precession, I will make use waveform model `NRSur7dq4`, which is the only model calibrated to numerical simulations of precessing BH binaries, including the full six-dimensional spin degrees of freedom and higher harmonics (Biscoveanu et al. 2021; Varma et al. 2019). The waveforms will be split randomly into training, development, and test sets with ratios of 0.8/0.1/0.1 respectively following how the network was trained previously. I will be computing mismatches between these waveforms using `PyCBC`, which requires optimization over both time and coalescence when the system is phase-on.

3.2. *Degeneracy Mapping*

Since it is difficult to infer component spin parameters from GW, we use $\chi_{\mathrm{eff}}, \chi_p$ to characterize the signal detected. The effective inspiral spin χ_{eff} is the mass-weighted average of the components of the BBH's spins aligned with the system's angular momentum

$$\chi_{\mathrm{eff}} = \frac{m_1 a_1 \cos\left(\theta_1\right) + m_2 a_2 \cos\left(\theta_2\right)}{m_1 + m_2} \quad -1 \leq \chi_{eff} \leq 1 \tag{2}$$

The effective precession spin can be modeled with a single parameter χ_p, which is defined to be the mass-weighted in-plane spin component that contributes to precession of the orbital plane at some (arbitrary) instant during the inspiral phase (Schmidt et al. 2015).

$$\chi_p = \max\left(a_1 \sin\left(\theta_1\right), \frac{4m_2^2 + 3m_1 m_2}{4m_1^2 + 3m_1 m_2} a_2 \theta_2\right) \quad 0 < \chi_p < 1 \tag{3}$$

The GW waveforms for systems with precession effects have richer morphology.

Expectedly, characterizing an extremely complex GW waveform with few parameters results in degeneracy in the parameter space (i.e. $\mathcal{MM} \ll 0.2$). Fig. 3a provides a visualization of the degeneracies between the parameters χ_{eff} and η in the case where BH spins are aligned with the orbital angular momentum.

I will be making similar plots with different parameters and reference data point. If time permits, I will investigate whether such degeneracies are broken at higher/different modes.

APPENDIX

WORK PLAN: UPDATED 07/07/2023

Before arrival: Start background reading on project. Download required software and libraries. Familiarize myself with the mismatch-prediction model by Ferguson (2023) and understand the working principles of the APIs used (TensorFlow and Keras).

Week 1-2: Orientation; Reproducing existing results. Write plotting routines. Investigate algorithms for identifying degeneracies in parameter space: Hessian (failed), KDE (in progress).

Week 3-4: Trip to LHO; report I.
Finish KDE plots and test Gaussian mixture for mapping out the parameter space.

Week 5-8: Compute mismatches for existing waveforms; report II. Prepare training data to retrain the network on high mass, precessing systems. Recover χ_{eff} relationship from the parameter space after smoothing. Map out degeneracies with precessing systems.

Week 9-10: Follow-up with interesting results; final report.

A. `MISMATCH.PREDICTION`

The training data for the published model at https://github.com/deborahferguson/mismatch_prediction consists of BBH systems with symmetric mass ratios $0.0826 \leq \eta \leq 0.25$ and spin magnitudes $0 \leq a_1, a_2 \leq 0.9695$ in various directions. For this project, we sample points in the parameter space where the mass ratio $1 \leq \frac{1}{q} \leq 10$ with $q \equiv m_2/m_1, m_2 \leq m_1$ and $0 \leq a_1, a_2 \leq 1$. At this stage of the project, we have not added in effects of procession ($\chi_p = 0$) and are considering systems where spin vectors are aligned or antialigned with the orbital angular momentum ($a_{1x} = a_{1y} = a_{2x} = a_{2y} = 0$). By setting the secondary mass of the system to 1, we have from equations 2, 3

$$\eta = \frac{q}{(q+1)^2} \tag{A1}$$

$$\chi_{\text{eff}} = \frac{qa_1 + a_2}{q+1} \tag{A2}$$

B. VISUALIZING DEGENERACIES

In the simplest case that we are considering, $\mathcal{MM}$ is an output from three input variables (q, a_1, a_2) when a reference point (q', a_1', a_2') is chosen in the three-dimensional parameter space. To map out the degeneracies, we sample 10,000 random points in the parameter for each reference point and assign a color map to the mismatch values associated with each point, as shown in fig. 1. By changing the coordinate system of the parameter space, we observe the approximate linear relationship between χ_{eff} and η as expected in fig. 2. In fig. 3, we were able to recreate the features of the degeneracy plot from (Ferguson 2023) subjected to differences in color scheme and figure scaling. We further experimented by varying the reference points along the line where the χ_{eff} and η are most degenerate. The resulting plot is shown in fig. 4, which maps very similar space. We further experimented with visualizing the degeneracies via fixing χ_{eff} and varying the initial mass and spin parameters in fig. 5. Mapping out contour lines on which the mismatch value is constant in the parameter space would be a reasonable direction to proceed.

C. IDENTIFYING CORRELATIONS

Initally, we attempted to identify the degeneracies in the parameter space by computing the eigenvalues of Hessians at each point. However, since the parameter space consists of discrete points, there is too much noise about the associated Hessian from interpolation. We then instead applied kernel density estimation (KDE) to smooth out the discretized parameter space by using `PESummary`. We set `weights =` $\mathcal{MM}$ and `method = "Reflection"` to account for the boundary conditions of the parameter space. The results for reference simulation at $\eta = 0.16, \chi_{\text{eff}} = 0$ are shown in fig. 6. As the KDE function only provides a visualization of the correlations, we plan to fit the injected simulations to a gaussian mixture model next, which would enable us to quantify the correlations between parameters.

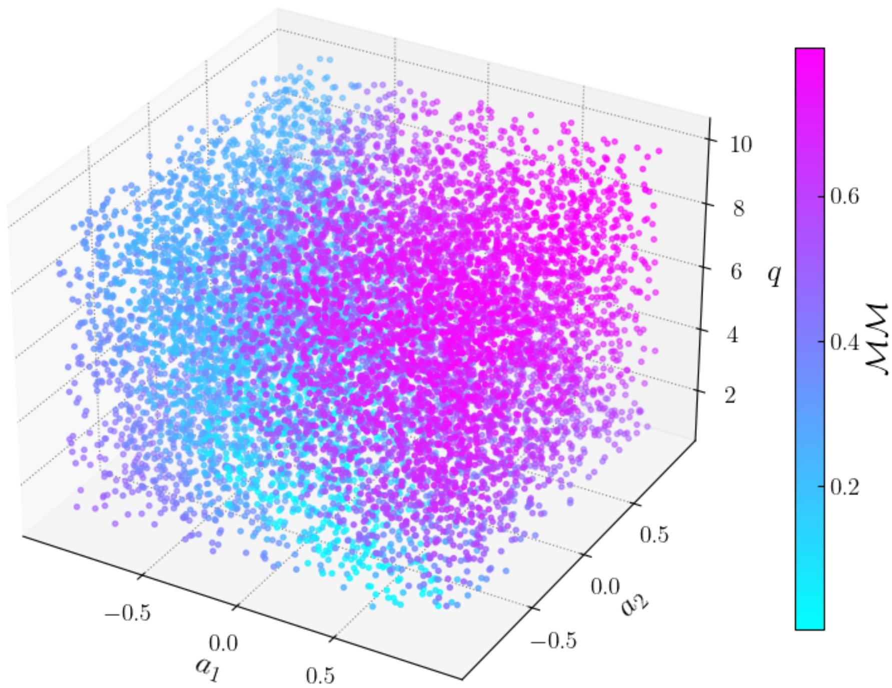


Figure 1: Scatter plot visualization of the three-dimensional parameter space with respect to reference simulation with $q = 1, a_1 = a_2 = 0$.

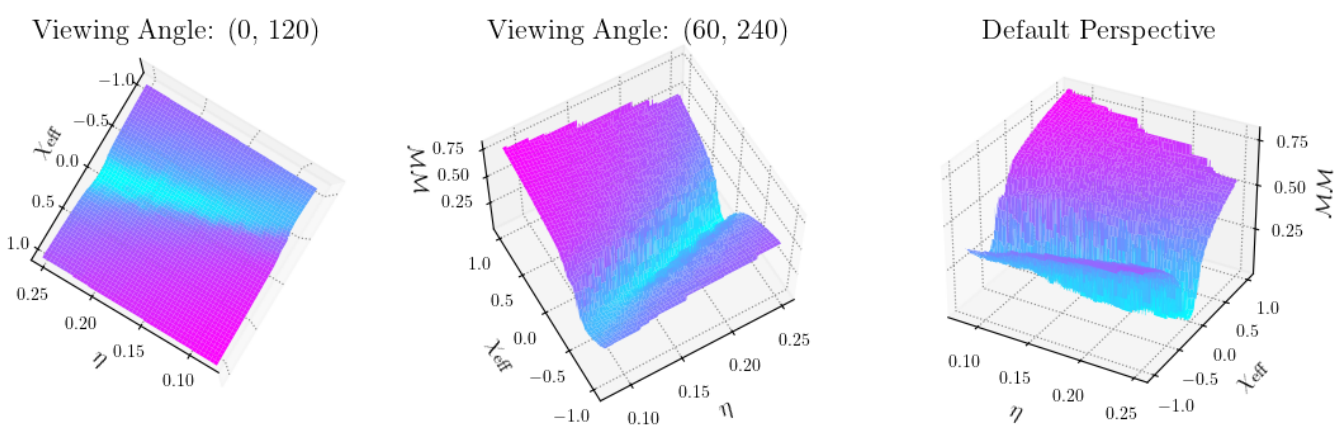
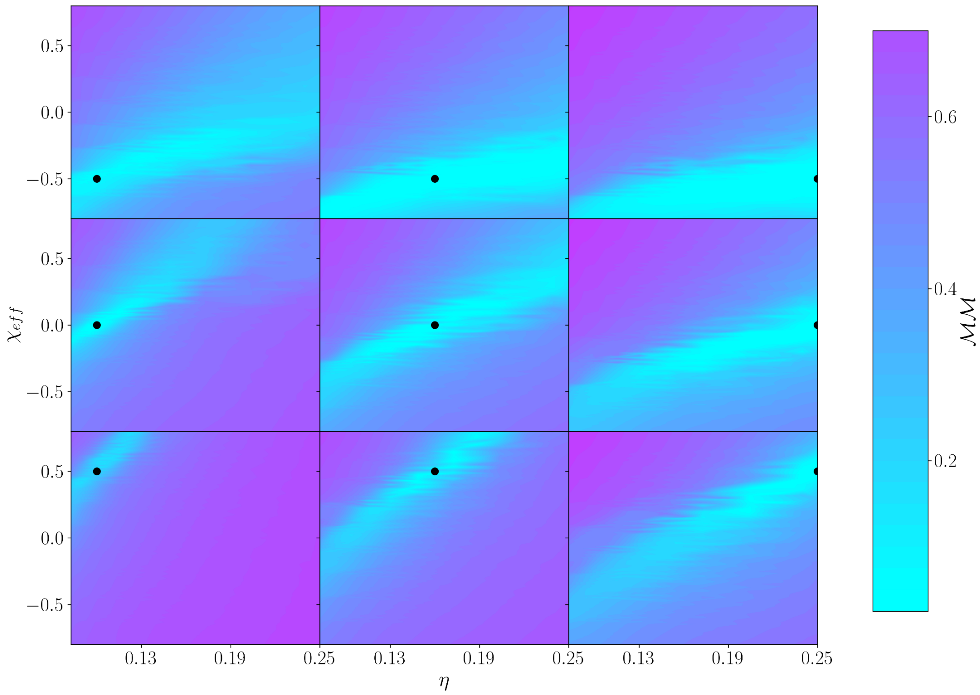
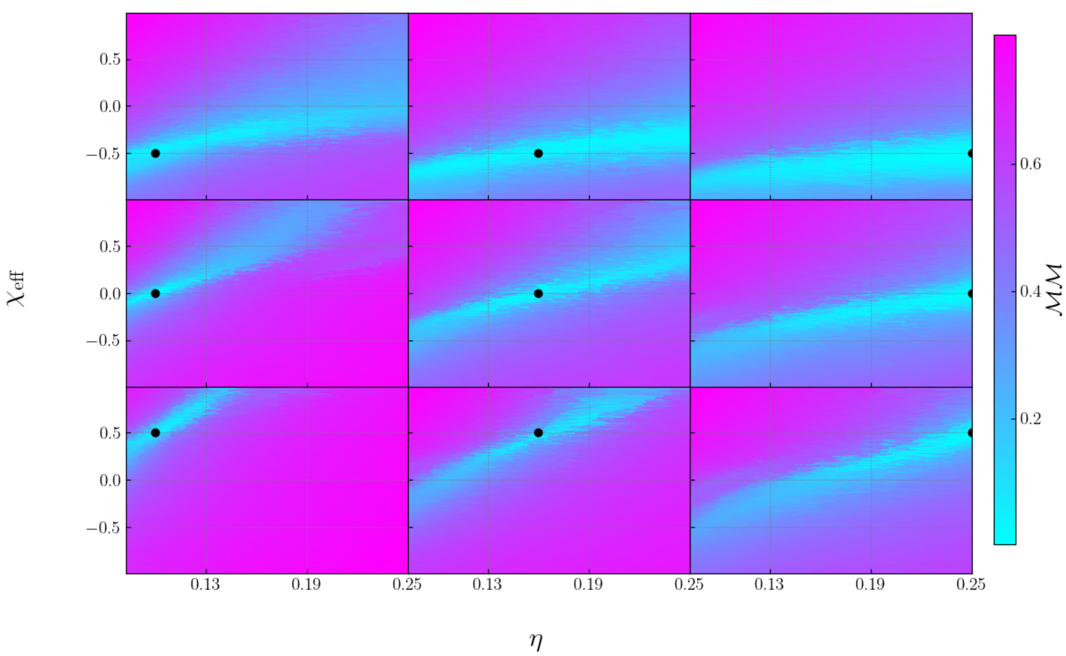


Figure 2: Mismatch between reference simulation at $\eta = 0.25, \chi_{\mathrm{eff}} = 0$ and sampled simulations throughout the parameter space.



(a)



(b)

Figure 3: Mismatch between waveform of reference binary (black dot) and of other binaries throughout the parameter space plotted as $\chi_{\rm eff}$ vs. η. From left to right, the reference simulation has $\eta = 0.1, 0.16, 0.25$. From top to bottom, the reference has $\chi_{\rm eff} = -0.5, 0, 0.5$. The original plot from Ferguson (2023) is shown in the figure above, and the recreation of the plot is shown below. Each subplot in the figure below is interpolated using the `nearest` method in `scipy.interpolate.griddata`.

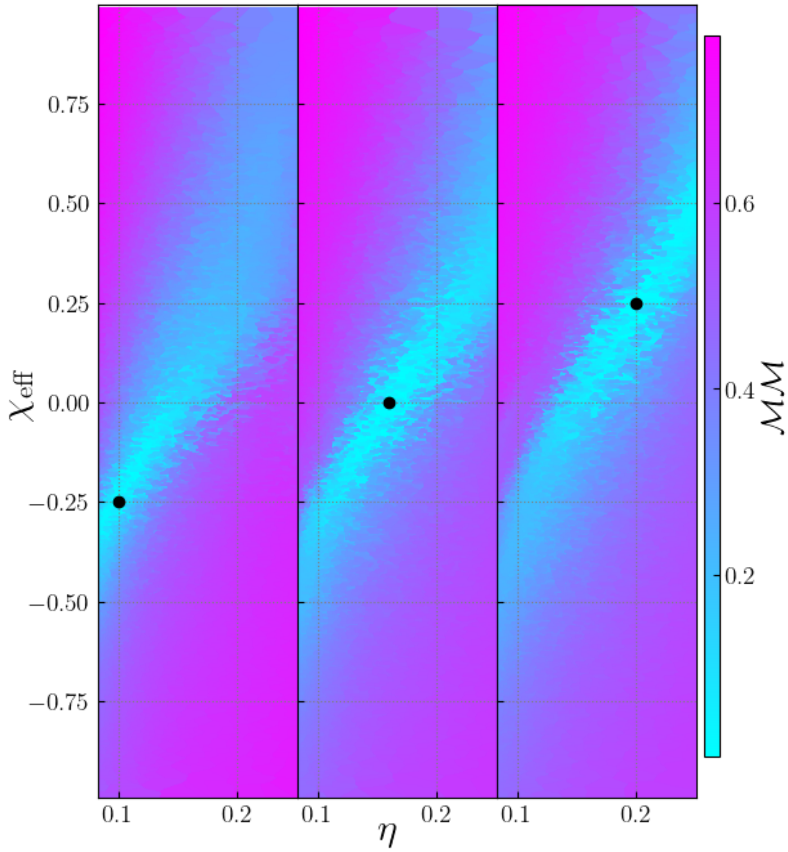


Figure 4: Reference simulations chosen at $\chi_{\rm eff} = -0.25, 0, 0.25$ and $\eta = 0.1, 0.16, 0.2$ along the line $\chi_{\rm eff} = 4.934\eta - 0.757$ where the two parameters are approximately degenerate.

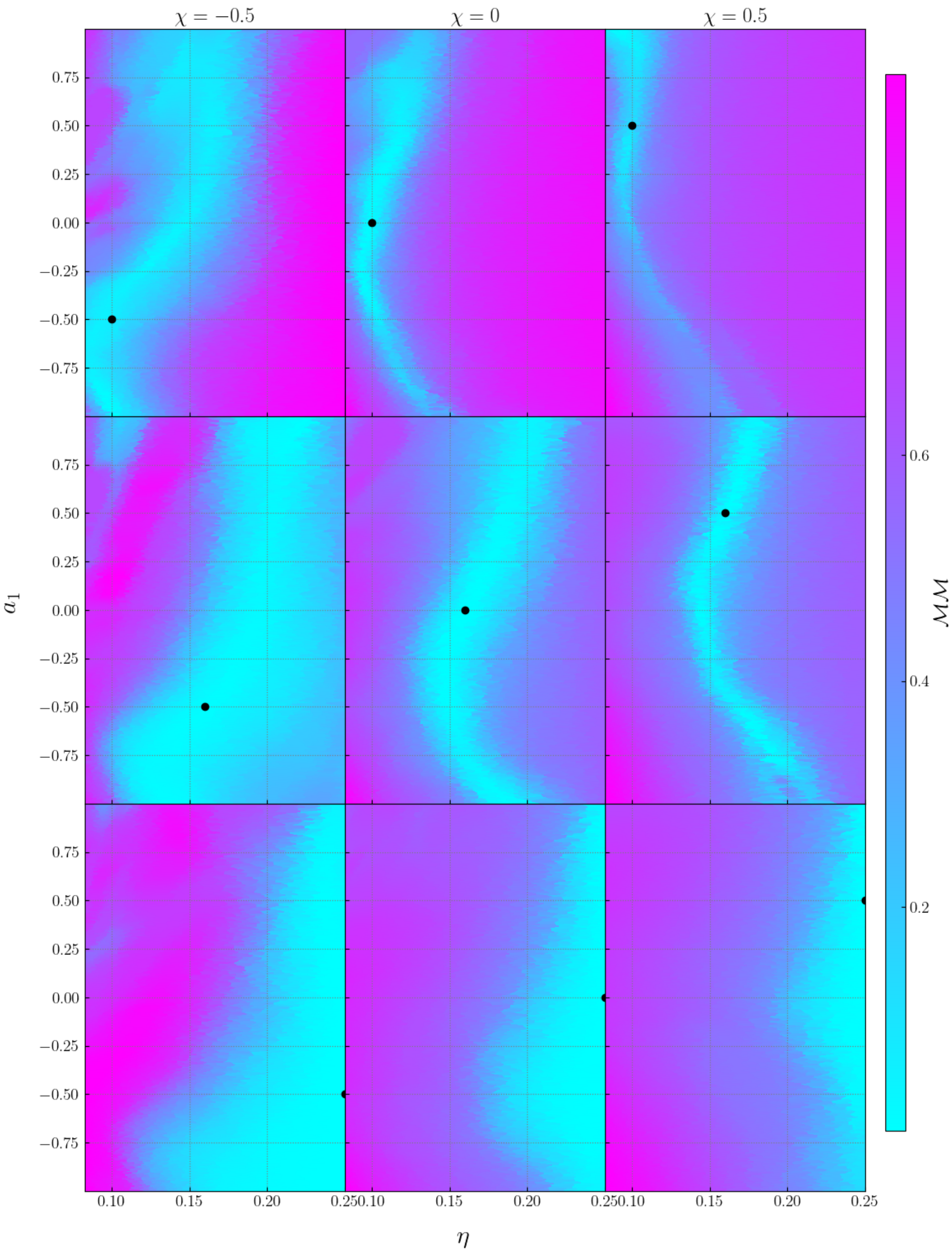
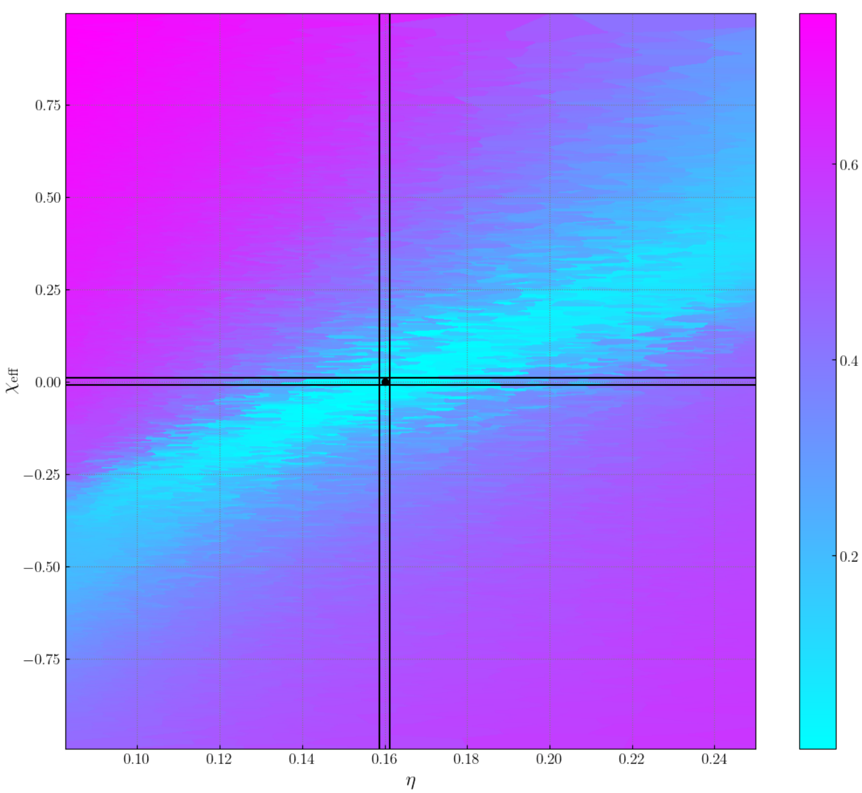
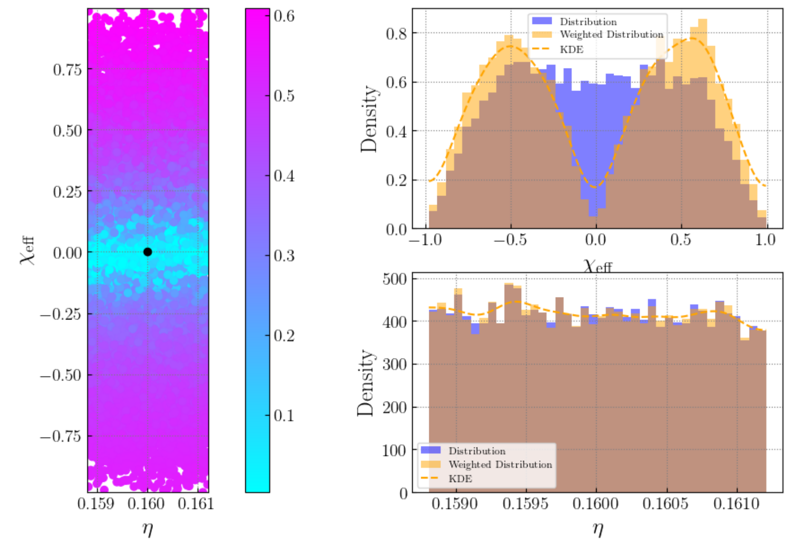


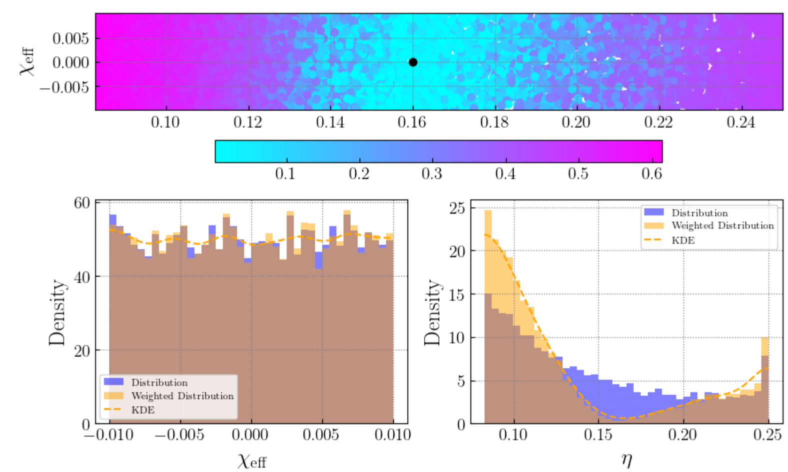
Figure 5: Mismatch between waveform of reference binary (black dot) and of other binaries throughout the parameter space plotted as a_1 vs. η while fixing the effective spin. From left to right, the effective spin is fixed at $\chi_{\text{eff}} = -0.5, 0, 0.5$. From top to bottom, reference has $\eta = 0.1, 0.16, 0.25$.



(a) $\eta = 0.16, \chi_{\mathrm{eff}} = 0$



(b) $\eta = 0.16 \pm 0.05$



(c) $\chi_{\mathrm{eff}} = 0 \pm 0.01$

Figure 6: 1-D KDE plot for slicing the parameter space along the horizontal and vertical

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