Using Multimessenger Synthesis to Constrain Core-collapse Supernova Distance and Orientation

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Abstract

Gravitational wave (GW) emission from the next nearby core-collapse supernova (CCSN) will be an important multimessenger, containing intrinsic properties of the CCSN and its progenitor star. Prior work has related expressions for the earlier GW "core-bounce" signal and later dominant frequency mode from the protoneutron star (PNS) to CCSN parameters like the rotation and density profile. Other work has shown similar relationships, including CCSN neutrino luminosities and mean energies. We run a set of 2D neutrino radiation-hydrodynamic CCSN simulations, via FLASH, to more precisely quantify relationships between neutrinos, GW observables, and CCSN properties: angular momentum and compactness. Likewise, we propose a new multimessenger-based method to break the existing degeneracy between CCSN distance and orientation, providing a constraint on both parameters in the event of a CCSN GW detection.

1 Background

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The effect of relativistic gravity in all stages of a core-collapse supernova (CCSN) is well known to be important - at the very least, the process of forming the proto-neutron star (PNS) via the star binding energy, and the compactness of the PNS itself mandates that we use general relativity when simulating such CCSNs. This is relevant in the context of GW astronomy, where CCSN are a point of interest due to their expected detection in future runs via the likes of LIGO, Virgo, and other gravitational wave (GW) observatories, as can be seen in Figure 1. More generally, understanding CCSN properties becomes relevant when considering its applications to other fields of physics - the conditions underlying that of a supernovae are difficult at best to reproduce in terrestrial laboratories, and hence these transients serve as insight into high-energy physical phenomenon [5]. Gravitational waves prove as a method of understanding this explosive phenomenon - it's well known that these signals are weakly coupled to matter, and hence if it were produced within a supernovae, it would pass through to us relatively unobstructed. It turns out that this property allows us to probe the internal structure of the progenitor CCSN (and the PNS itself). Having reliable general relativistic simulations of CCSN, then, becomes vital for astronomers working in the field. FLASH is one such simulation software capable of taking into account supernovae processes (hydrodynamics, nuclear reaction networks, etc.) in the

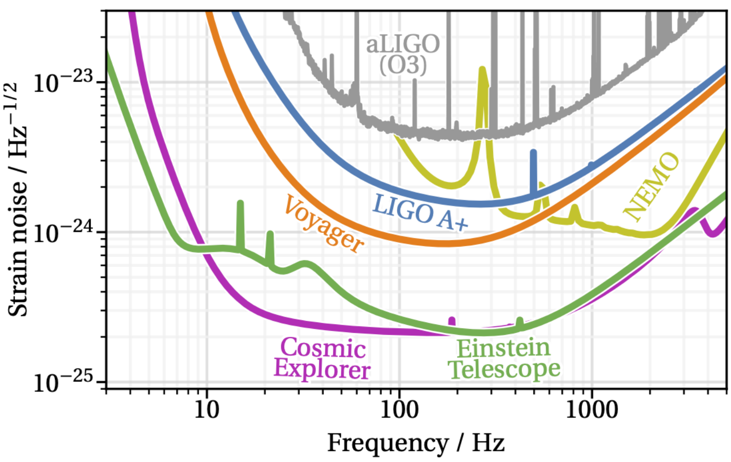

Figure 1: Ligo sensitivity curve.

context of general relativity. A case of possible general relativity (GR) approximation in FLASH, for example, is the effective GR potential detailed in [1], where the (on first principle) standard Newtonian gravitational potential

$$\phi(r) = -4\pi \int_0^\infty dr' r'^2 \frac{\rho}{|r - r'|} \tag{1}$$

is replaced with the TOV potential

$$\phi(r) = -4\pi \int_r^\infty \frac{dr'}{r'^2}(\frac{m_{TOV}}{4\pi} + r'^3(P + p_v)) \times \frac{1}{\Gamma^2}(\frac{\rho + e + P}{\rho}) \tag{2}$$

where ρ, $e = \rho\epsilon$, and P are rest-mass density, internal energy density, and gas pressure, respectively. In principle, one is able to extract relevant parameters from FLASH simulations to perform an analysis on CCSNe. However, it must be noted that there exists degeneracy when one only relies on GW data.

1.1 Intrinsic Variables

The four general parameters of interest for CCSN are distance (D), viewing angle (θ), rotation profile, and compactness. The dimensionless compactness parameter is a measure of pre-SN core structure, defined by [3] to be

$$\xi_M = \frac{M/M_\odot}{R(M_{bary} = M)/1000km}|_{t_{bounce}} \tag{3}$$

Here $R(M_{bary} = M)$ is a radial coordinate around baryonic mass M at time of core bounce. Generally, the choice of M ranges from $1.5 - 2.5M_\odot$. We pick 1.75, for similar reasons as in [3]. [5] in simulations defines a physically motivated rotation profile as

$$\Omega(r) = \Omega_0[1 + (\frac{r}{A})^2]^{-1} \tag{4}$$

Here r is a radial coordinate, Ω_0 is central angular speed, and A is a differential rotation parameter. [4] notes a linear relation between the compactness parameter and differential rotation parameter A. Hence we represent the rotation profile solely with the variable Ω_0

1.2 Observables

These parameters of interest must be linked to variables that Earth-based observers can feasibly detect. Restricting our attention to solely gravitational waves, as [5] details, we can extract GW signals from simulations with the dominant quadropole moment formula via slow motion, weak-field formalism, yielding a GW strain that goes as

$$h_+ \approx \frac{3}{2}\frac{G}{Dc^4}\frac{d^2 I_{zz}}{dt^2} sin^2\theta \tag{5}$$

Here G is the gravitational constant, c is the speed of light, and I_{zz} is the reduced-mass quadrupole moment. Analyzing this strain in prior simulations leads to two features of interest, the first of which is the GW bounce signal - a burst of GWs produced at the point where nuclear forces stop matter infall post iron-nuclei photodissociation. As detailed in [6], this bounce signal can be approximated as

$$\Delta h \approx \frac{GMR^2\Omega_0^2}{c^4 D} \sin^2\theta \tag{6}$$

The second notable feature is that of the peak GW frequency. In the case of FLASH simulations, [5] notes that a semianalytic formula (slightly modified from its original derivation in [2]) for this goes as

$$f_{peak} \approx \frac{1}{2\pi}\frac{GM}{R^2 c}\sqrt{2.1\frac{m_n}{\langle E_{\overline{\nu}_e}\rangle}}(1 - \frac{GM}{Rc^2})^2 \tag{7}$$

Here M and R are PNS mass and radius, respectively. m_n is neutrino mass and $\langle E_{\overline{\nu}_e}\rangle$ is mean electron-type antineutrino energy. Evidently, these two equations are not nearly enough to constrain all four intrinsic parameters to any great degree of success (in fact, even if one restricts themselves to just distance and orientation, there exists a clear degeneracy in these equations). As the prior equation implies, and from the general ability of neutrinos to transmit unimpeded from stellar cores to Earth-based observatories like Super-Kamiokande, we also include neutrino counts and neutrino energy. As [8] shows, it turns out these two observable parameters can be correlated to stellar compactness, the GW "ramp-up" slope (the slope of peak GW frequency), and distance.

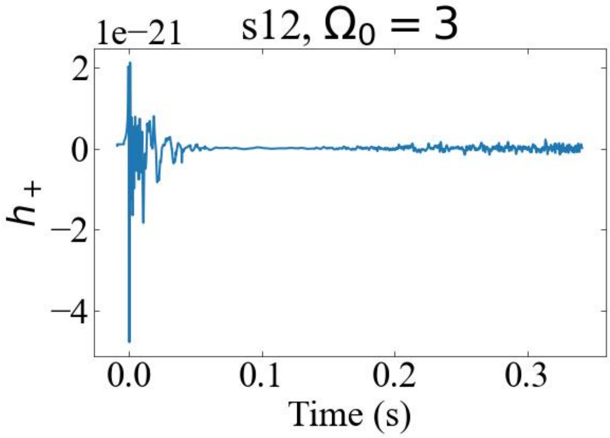

Figure 2: Plot of dimensionless GW strain versus time for the 12 $M_\odot$, $\Omega_0 = 3$ simulation.

2 Methods

The idea of the project is to take currently existing multidimensional CCSN datasets from FLASH simulations to extract the four aforementioned observables (bounce amplitude, ramp-up slope, neutrino counts, and neutrino energy). In principle, these four observables can be synthesized to break the distance/orientation degeneracy in the FLASH simulations and provide constraints in the event of a CCSNe detection. The general approach is to follow similar methodology as these aforementioned papers - in particular, [8] and [5] imply that a linear fit captures to good order the relationships between several of these variables.

2.1 Simulations

The simulations are all run via FLASH, using Newtonian hydrodynamics with an effective GR gravitational potential (GREP). Neutrino transport is modeled through an M1 scheme, detailed further in [7]. The neutrino types to be focused on are electron-type neutrinos (nue), electron-type antineutrinos (anue), and heavy type neutrinos (nux). In total, we use 19 2D CCSNe with the SFHo equation of state (EOS). The simulations have zero age main sequence (ZAMS) mass ranging from 12, 20, 40, and 60 $M_\odot$, with a central rotation rate Ω_0 ranging from 0, 0.5, 1, 2, and 3 rad/s (the 40 $M_\odot$, $\Omega_0 = 3$ is not physical, hence we end with 19). From these simulations, the quantities of interest we directly extract are those of total luminosity w.r.t each neutrino type, total average and rms energy w.r.t each neutrino type, and the quadrupole moment of the system, all as functions of time. Here we assume that the neutrino emission from our CCSN simulations are isotropic in nature (ie. no angular dependence). The compactness parameters for each simulation were independently extracted from ZAMS-mass pre-SN models, which were calculated using KEPLER code. This was done via spherical numerical integration over the radial stellar density function for each progenitor model from $r = 0$ up to the radius corresponding to a total mass of 1.75 $M_\odot$.

2.2 Gravitational Waves

We use second-order finite differencing on the mass quadrupole moment in the simulations to obtain dimensionless strain h_+ as a function of time. Doing so yields a function that looks like, for example, Figure 2. Immediately we can confirm the presence of a non-zero bounce amplitude from the time-domain alone. This is evident from the initial large "spike" in amplitude seen in Figure 2. Hence we extract the bounce signal, following methodology outlined in [5]. To observe the peak GW frequency and thereafter the ramp-up slope, we convert the dimensionless strains to the frequency domain via a spectrogram analysis. Doing so yields the expected linear ramp-up slope, as seen in Figure 3. With the existence of the key CCSNe GW properties confirmed, we follow the methodology outlined in section 3.4 in [5] to obtain the values of the ramp-up slopes ($\dot{f}$) in all simulations.

2.3 Neutrinos

As aforementioned, we obtain information for anue, nue, and nux luminosities and energies as functions of time. We note from the example shown in Figure 4 that there is a clear peak in this luminosity. This is true for all neutrino types across all simulations. It is known that CCSN neutrino emission is emitted primarily from

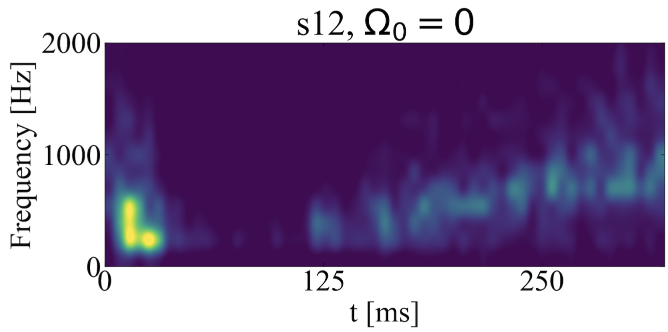

Figure 3: Spectrogram plot of dimensionless GW strain signal for the 12 $M_\odot$, $\Omega_0 = 0$ simulation.

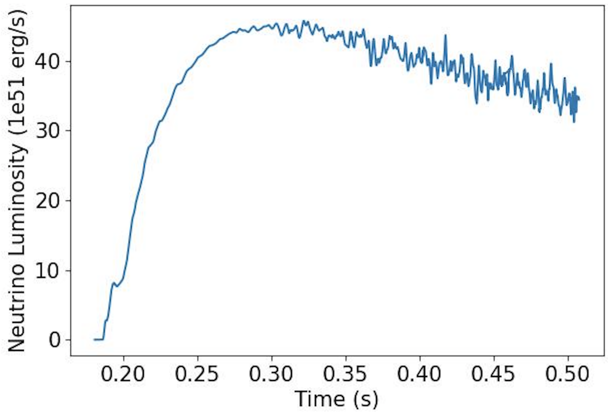

Figure 4: Anue luminosity as a function of time for the 12 $M_\odot$, $\Omega_0 = 0$ simulation.

the core, and is hence related to the dimensionless compactness parameter $\xi_{1.75}$. Furthermore, the standard classical effect of spherical "flattening" due to higher rotation rates implies that more neutrinos will be emitted further from the center as Ω_0 increases, implying luminosities with generally lower temperatures. This in turn implies that there exists some correlation between Ω_0 and peak neutrino luminosity. As [8] implies, this relation is likely linear. Testing this across all simulations confirms this linear relationship, as evident in Figure 5.

2.3.1 Snowglobes

Peak luminosity immediately connects an observaable to two intrinsic parameters. We continue our neutrino analysis using SNOwGLoBES, an open source python package capable of computing interaction rates and distributions of neutrino energies and counts (N_{tot}) given luminosity information. We assume a fiducial distance of 10 kpc between our CCSNe simulations and the super-Kamiokande detectors to obtain these values. Doing so gives us four (in principle) observable quantities for each simulation (Δh, $\dot{f}$, antineutrino peak luminosity $L_{peak}^{\overline{\nu}_e}$, and N_{tot}).

3 Results

We then construct a general linear fit over three equations with our intrinsic and extrinsic variables, with the specific variables in each equation motivated by prior classical arguments and the papers [5], [8]. Doing so connects GW observables to CCSN physics, yielding the following equations:

$$\xi_{1.75} = 1.42\Delta h + 7.73\dot{f} - 1.30 \tag{8}$$

$$\xi_{1.75} = (4.00)N_{tot} \times \frac{(100\text{kpc})^2}{D^2} + (-2.97)\dot{f} + (0.399) \tag{9}$$

$$L_{peak}^{\overline{\nu}_e} = (-14.1)\Omega_0 + (42.6)\xi_{1.75} + 48.2 \tag{10}$$

Here Δh is scaled by 1×10^{-20}, $\dot{f}$ is scaled by 1×10^4 (units of Hz/s), N_{tot} is scaled by 1×10^4, Ω_0 is in rad/s, and $L_{peak}^{\overline{\nu}_e}$ is scaled by 1×10^{51} ergs/s.

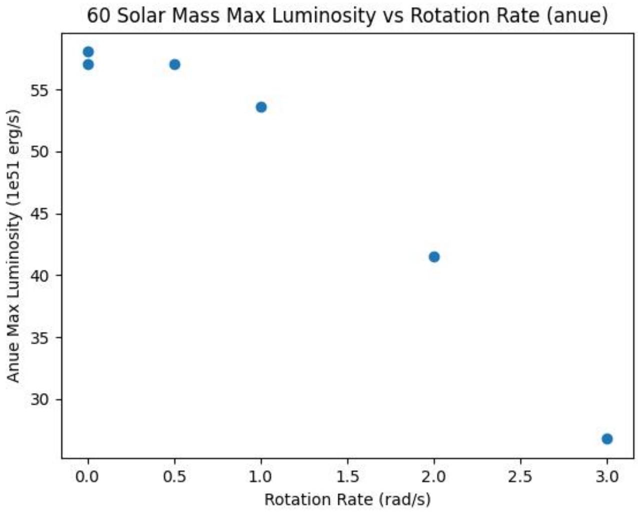

Figure 5: Peak anue luminosity for each 60 $M_\odot$ simulation plotted over central rotation rate Ω_0.

Coefficients	p-statistic
-1.30277	0.00161702
7.72774	0.0000755425
1.42308	0.0000493404
0.398785	0.00115819
3.99562	0.000308966
-2.96515	0.00361336
48.1613	0.00000557669
-14.0466	0.0000440853
42.5503	0.000871774

Table 1: Fit statistic for each (unrounded) coefficient in the three linear fits.

We also find that the p-values for each coefficient in all linear fits are generally less than order 1×10^{-3}, implying a generally accurate fit, and agreement with accuracy results in [8] - as can be seen in Table 1. In principle, equations 8, 9, and 10 can be combined with equation 6 and solved given adequate Super-Kamiokande and LIGO detector data for important properties of the CCSNe itself.

4 Conclusion/Future Work

We have thus provided a heuristic with which one can find bounds on CCSNe compactness, viewing angle, distance, and rotation in the event of a detection. This work of course opens up further modes of investigation. Some primary directions involve the relaxing of assumptions. For example, we assume isotropy in neutrino emissions, and thus no θ dependence in equation 9. It is known ([5]) that for rotating CCSN, the neutrinosphere becomes deformed. This resulting non-spherical emission profile contains further information about the orientation of the CCSN with respect to an observer on Earth. Hence a further analysis on constraining these four variables would need to take into account directionally dependent neutrino luminosity calculations. These simulations could also be generalized. A current objective being worked toward is that of expanding the parameter space - our simulations cover a relatively small subset of potential ZAMS-mass CCSNe and rotation profiles. More simulations run will inevitably provide better bounds on any such fit. One could also run full 3D simulations, or vary the equation of state. There are also likely exists further relationships between other CCSNe observables. One promising observable is that of electromagnetic signatures. It is possible that there exists unquantified correlations between electromagnetic signatures and GWs. If we run FLASH simulations through hydrodynamic code (e.g., Sedona), light curve relationships with respect to both neutrino and GW quantities may be revealed.

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